Short Communication: Optimal Random Regression Models for Milk Production in Dairy Cattle

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ABSTRACT

Legendre polynomials of orders 3 to 8 in random regression models (RRM) for first-lactation milk production in Canadian Holsteins were compared statistically to determine the best model. Twenty-six RRM were compared using LP of order 5 for the phenotypic age-season groupings. Variance components of RRM were estimated using Bayesian estimation via Gibbs sampling. Several statistical criteria for model comparison were used including the total residual variance, the log likelihood function, Akaike's information criterion, the Bayesian information criterion, Bayes factors, an information-theoretic measure of model complexity, and the percentage relative reduction in complexity. The residual variance always picks the model with the most parameters. The log likelihood and information-theoretic measure picked the model with order 5 for additive genetic effects and order 7 for permanent environmental effects. The currently used model in Canada (order 5 for both additive and permanent environmental effects) was not the best for any single criterion, but was optimal when considering all criteria.

Key words: random regression model, optimization, statistical criteria

Random regression models (RRM) using Legendre polynomials (LP) have been applied to production traits in dairy cattle, and to growth traits in beef cattle and swine. For dairy cattle the regressions are on number of DIM for both additive genetic (ADD) and permanent environmental (PE) effects. The order of the LP in the RRM is important in that estimates of genetic parameters can differ with the order (Misztal et al., 2000). The choice of submodel for ADD or PE is the focus in finding an optimal RRM. Jamrozik et al. (1997) found that an LP of order 5 for ADD and PE had a slight advantage in terms of prediction error variances for daily yield,

compared with LP of order 3. Commonly, RRM with orthogonal polynomials outperform models with lactation curve functions having the same number of parameters for ADD and PE effects (Jamrozik and Schaeffer, 2002). Furthermore, orthogonal polynomials have reduced correlations among the estimated coefficients (Schaeffer, 2004). Meyer (2000) and Pool et al. (2000) showed that the orders of the orthogonal polynomials do not need to be equal for ADD and PE effects. Guo and Schaeffer (2002) compared 18 submodels using several statistical criteria and concluded that orthogonal polynomials of order 4 or 5 were appropriate for RRM. In previous studies on optimization of RRM, the orders of LP fitting ADD and PE effects were usually limited to less than 5 and the statistical criterion for the choice of models was not uniform. Besides random regressions, functions are needed to account for the phenotypic relationship between test-day records and DIM (Schaeffer, 2004), and these are usually nested within age and season of calving. The objective of this study was to expand the search for an optimal RRM model to LP of order 3 to 8.

After edits, data were 90,023 first-lactation test-day milk yields on 10,130 Canadian Holstein cows calving from 1988 to 1999. Data were restricted to DIM between 5 and 330 d inclusive, to test-day milk yields between 1.5 and 90 kg inclusive, to the number of records per cow greater than 7, and to the number of records within a herd test date greater than 10. The number of herd-test day subclasses was 7,657. Pedigrees were traced back 3 generations or to unknown parents.

The single-trait RRM (Schaeffer, 2004) for test-day milk yields was

$$y_{ijkt} = HTD_i + g(t)_j + r(a,t,m_1)_k + r(pe,t,m_2)_k + e_{ijkt}$$

where y_{ijkt} is milk yield on the k th animal at DIM t belonging to the i th herd-test date subclass, and the j th age-season grouping (age at calving, 4 groups, by season of calving, 3 seasons); HTD_i is the effect of the herd-test date on test-day records taken on the same date in a herd; $g(t)_j$ are LP of order 5 that account for

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the phenotypic trajectory of the average observations across all animals belonging to the j th age-season of calving grouping; $r(a, t, m_1)$ and $r(pe, t, m_2)$ are submodels (LP) for ADD and PE effects, respectively, of order m , the number of covariables related to time; and e_{ijkt} is assumed to follow a normal distribution with mean null and variance $\sigma_{e(t)}^2$. Different residual variances were allowed for 30 time intervals within the interval from 5 to 330 DIM, defined as 5–20 DIM, 21–30 DIM, . . . , 281–290 DIM, 291–305 DIM, and 306–330 DIM. Residual effects were uncorrelated both within and between individuals.

The 2 random regression effects, ADD and PE, were characterized using LP of different orders, which ranged from 3 to 8. The models were designated by LP_{mn}, where m is the order for the ADD effects and n is the order for the PE effects. Thus, LP35 is a model with LP of order 3 for ADD and of order 5 for PE effects; a total of 26 RRM were compared. The choice of optimal RRM was based on statistical criteria. The following statistical criteria were used:

$$\text{Total residual variance (TRV)} = \sum_{t=5}^{330} \sigma_{e(t)}^2;$$

$$\text{Log likelihood function } [\mathbf{log(L)}] = -2\log(ML_k);$$

$$\text{Akaike's information criterion (AIC): } AIC_k = -2\log(ML_k) + 2p_k \text{ (Akaike, 1973);}$$

$$\text{Bayesian information criterion (BIC): } BIC_k = -2\log(ML_k) + p_k \log(n) \text{ (Schwarz, 1998);}$$

$$\text{Information-theoretic measure of model complexity (ICOMP): } ICOMP_k = -2\log(ML_k) + C_1(\Sigma) \text{ (Bozdogan, 2000); and}$$

$$\text{Percentage relative reduction of complexity (PRRC): } PRRC_k = [(C_1(\Sigma_k) - C_1(\Sigma_{kR})) / C_1(\Sigma_k)] \times 100\% \text{ (Bozdogan, 2000),}$$

where p_k is the number of free parameters in model k ; n is the number of observations that contribute to the likelihood; Σ_k and Σ_{kR} are covariance and correlation matrices of the parameters of model k , and $C_1(\Sigma) = \text{rank}(\Sigma) \log[\text{trace}(\Sigma) / \text{rank}(\Sigma)] - \log(|\Sigma|)$.

Bayesian factor (BF): $BF_{01} = p(y|M = M_0) / p(y|M = M_1)$ (Kass and Raftery, 1995), provides a contrast of model M_0 against model M_1 , where $p(y|M = M_k)$ is an integrated (marginal) likelihood. According to Kass and Raftery (1995), a BF value greater than 150 [or $2\log(BF_{01})$ greater than 10] indicates very strong evi-

dence in favor of model M_0 . Generally, small values are favorable for each statistic except for PRRC and BF [or $\log(BF)$], where large values are favored. For BF, the model LP65 was used for M_1 .

The covariance matrices of additive genetic, permanent environment random regression coefficients and residual variances of all competing RRM were estimated using the GIBBS package of DMU (Madsen and Jensen, 2000).

The results under each criterion are expressed as differences from the model giving the smallest value for TRV, $\log(L)$, AIC, BIC, and ICOMP (Table 1). The results for $\log(BF)$ are based on comparing each model to LP65, and PRRC is a percentage value where the higher value is preferred. Total residual variance decreased within the orders for the ADD as the order of the PE effects increased. The difference among all competing models was nonsignificant according to likelihood ratio tests. Based on AIC and BIC, the model with the least number of estimated parameters (LP33) was the best. Model LP57 performed best on $\log(L)$ and ICOMP, whereas LP65 and LP66 did best on $\log(BF)$ and PRRC, respectively. There was not unanimous agreement on the best submodel among the 7 comparison methods used. In general, models were improved when the order of LP for PE was higher than for the order of ADD.

Because each statistical criterion indicated a different model as being optimal, the problem becomes one of deciding upon a statistical criterion that is best for comparison purposes. There are advantages and disadvantages of each criterion. Therefore, an index was created that added together TRV, $\log(L)$, the absolute value of $\log(BF)$, AIC, BIC, ICOMP, and the negative of PRRC. The numbers are shown in the last column of Table 1 under INDEX. The index shows that model LP55 gave the smallest value, and therefore was the optimal model. Model LP55 is the model used in practice (in Canada) based on subjective assessments of researchers. Perhaps a better weighting of the different criterion would give a better index for comparison, but eventually a choice has to be made. Usually, only 1 criterion would be used to compare models rather than 7 because of the work involved in computing. Given that the model used in Canada is actually a multiple-trait model with 3 lactations and 4 traits per lactation, using a model with orders greater than LP55 may not be computationally feasible.

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Table 1. Model selection criteria of the competing random regression models with LP5 for age-season of calving grouping and different orders of Legendre polynomials (LP) for additive and permanent environment effects, respectively

Model	Selection criteria ¹							
	TRV	log(L)	log(BF)	AIC	BIC	ICOMP	PRRC (%)	INDEX ²
LP33	120.5	12.7	-94.2	0.0	0.00	24.4	61.28	190.5
LP34	84.8	19.5	-198.7	14.8	26.6	38.3	61.21	321.5
LP35	61.1	14.1	-141.5	19.4	46.0	27.6	59.23	250.5
LP36	43.5	6.7	-108.4	24.0	68.3	12.9	59.13	204.7
LP37	21.4	6.4	-52.9	37.6	102.6	12.4	54.24	179.1
LP43	92.9	10.3	-244.5	5.6	17.4	19.9	58.45	332.2
LP44	83.5	9.5	-135.8	12.8	36.4	18.6	67.61	229.0
LP45	59.4	7.5	-92.7	20.8	59.2	14.7	65.61	188.7
LP46	41.9	5.7	-66.5	31.0	87.1	11.2	64.23	179.2
LP47	19.1	4.3	-128.2	43.6	120.4	8.6	60.06	264.1
LP53	73.7	16.1	-152.1	21.4	48.0	31.6	55.73	287.2
LP54	65.9	10.8	-209.5	24.1	62.5	21.2	50.77	343.2
LP55	51.6	5.0	-19.3	28.3	81.4	9.8	62.35	133.1
LP56	39.3	5.2	-46.7	40.4	111.3	10.3	68.82	184.4
LP57	17.2	0.0	-1.5	49.2	140.8	0.0	64.44	144.3
LP63	59.8	13.1	-167.5	30.4	74.7	25.7	59.51	311.7
LP64	52.3	5.2	-197.1	30.5	86.6	10.2	50.89	331.0
LP65	42.9	15.4	0.0	50.6	121.5	30.7	70.63	190.5
LP66	35.7	13.1	-58.2	60.4	149.0	26.5	73.69	269.2
LP67	14.9	2.8	-1.3	63.4	172.7	4.5	70.28	189.3
LP73	43.8	6.0	-125.7	37.3	102.2	11.6	58.58	268.0
LP74	35.4	9.5	-119.9	48.7	125.5	18.8	64.27	293.5
LP75	27.9	6.8	-55.2	56.1	147.7	13.7	69.15	238.2
LP76	20.8	1.4	-41.5	62.7	172.0	3.1	72.73	228.8
LP77	11.3	8.8	-5.7	84.0	214.0	17.9	72.39	269.3
LP87	0.0	9.8	-80.1	101.1	254.7	20.1	71.39	394.4

¹TRV = Total residual variance; log(L) = log likelihood function; AIC = Akaike's information criterion; BIC = Bayesian information criterion; ICOMP = Information-theoretic measure of model complexity; PRRC = percentage relative reduction of complexity; TRV, log(L), AIC, BIC, and ICOMP are relative to their minimum values, respectively.

²INDEX = TRV + log(L) + abs[log(BF)] + AIC + BIC + ICOMP - PRRC.

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