Valuation of Milk Composition and Genotype in Cheddar Cheese Production Using an Optimization Model of Cheese and Whey Production

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ABSTRACT

A mass balance optimization model was developed to determine the value of the κ -casein genotype and milk composition in Cheddar cheese and whey production. Inputs were milk, nonfat dry milk, cream, condensed skim milk, and starter and salt. The products produced were Cheddar cheese, fat-reduced whey, cream, whey cream, casein fines, demineralized whey, 34% dried whey protein, 80% dried whey protein, lactose powder, and cow feed. The costs and prices used were based on market data from March 2004 and affected the results. Inputs were separated into components consisting of whey protein, ash, casein, fat, water, and lactose and were then distributed to products through specific constraints and retention equations. A unique 2-step optimization procedure was developed to ensure that the final composition of fat-reduced whey was correct. The model was evaluated for milk compositions ranging from 1.62 to 3.59% casein, 0.41 to 1.14% whey protein, 1.89 to 5.97% fat, and 4.06 to 5.64% lactose. The κ -casein genotype was represented by different retentions of milk components in Cheddar cheese and ranged from 0.715 to 0.7411 kg of casein in cheese/kg of casein in milk and from 0.7795 to 0.9210 kg of fat in cheese/kg of fat in milk. Milk composition had a greater effect on Cheddar cheese production and profit than did genotype. Cheese production was significantly different and ranged from 9,846 kg with a high-casein milk composition to 6,834 kg with a high-fat milk composition per 100,000 kg of milk. Profit (per 100,000 kg of milk) was significantly different, ranging from \$70,586 for a high-fat milk composition to \$16,490 for a low-fat milk composition. However, cheese production was not significantly different, and profit was significant only for the lowest profit (\$40,602) with the κ -casein genotype. Results from this model analysis showed that the optimization model is useful for determining costs and prices

for cheese plant inputs and products, and that it can be used to evaluate the economic value of milk components to optimize cheese plant profits.

Key words: cheese yield prediction, κ -casein, milk fat, optimization model

INTRODUCTION

Reducing production costs is an important contribution to increasing profitability. An important factor influencing production costs is the yield of cheese from a quantity of milk. Increasing the yield will positively influence both fixed costs and the cost of goods per unit of product. To evaluate the net economic benefit of increasing the frequency of an allele (A or B for κ -CN) before embarking on a breeding program, a linear program was created to determine the optimal contribution of various inputs to maximize profit. The objectives of this study were to develop a mass balance linear program to estimate cheese, whey, and whey product yields for different milk compositions and κ -CN genotypes and to evaluate the dollar value of milk fat, milk protein, and the κ -CN genotype to the cheese-production facility. The model evaluates the effects of milk composition and genotype (κ -CN AA or BB) on cheese yield.

The genetic variants of the milk protein κ -CN have been shown to have an important influence on the composition and cheese-making properties of milk (Feagan, 1979; Schaar, 1985; Aleandri et al., 1990; Ng-Kwai-Hang, 1993; Tong et al., 1994). A significant positive effect on cheese yield has been reported for different genotypes of the proteins (Marziali and Ng-Kwai-Hang, 1986; Aleandri et al., 1990; Ng-Kwai-Hang, 1993; Tong et al., 1994), where the increase in cheese yield ranged from 2 for the κ -CN AA allele to about 8% for the κ -CN BB allele. Because cows can be genotyped for these proteins by DNA analysis of either a blood or a milk sample, the opportunity exists to breed for a future milk supply that possesses superior manufacturing properties.

General Description of the Model

The model plant was an integrated manufacturer capable of producing several products, including Cheddar cheese, cream, fat-reduced whey, whey cream, deminer-

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Figure 1. Flow of milk and milk products to cheese and whey products. Ovals are inputs into the cheese-making process; hexagons are retentions; rectangles are intermediates; diamonds are final products sold. WC = Whey cream; FRW = fat-reduced whey; DM = demineralized; WPC = whey protein concentrate; PRO = protein; WPC 34% PRO = whey protein concentrate 34% protein; WPC 80% PRO = whey protein concentrate 80% protein.

alized whey, whey protein concentrates at 34 and 80% protein, lactose powder, CN fines, and cow feed. Depending on the price and costs of cream, cream can be skimmed from milk and sold, or it can be bought to increase Cheddar cheese production. Whey cream produced from reducing whey to fat-reduced whey can also be sold or recycled back into cheese production. Figure 1 shows a flow diagram of inputs, intermediate products, and output products. Retentions are the proportion of inputs retained in a product and can vary with the plant and the genotype of the cow producing the milk. Permeate refers to the leftover components after products have been made. Lactose powder and cow feed are produced from permeate. However, because compositions and prices were difficult to obtain for lactose pow-

der and cow feed, the results relating to their production are not discussed.

Based on the model plant, a single-period linear programming model was developed using general algebraic modeling (GAMS) system software (Brooke et al., 1998) with BDMLP (Brooke, Drud, and Meeraus linear program) as the solver. The BDMLP solver is a linear program and mixed integer program solver that is provided with the GAMS system. The model tracks 6 milk components (whey protein, CN, fat, lactose, ash, and water) of each input (milk, cream, NDM, condensed skim milk, and starter and salt) and allocates them to intermediate and final products based on retention rates and economic values. Plant capacity was 100,000 kg of milk daily.

The purchase and replacement of capital equipment required to operate a cheese plant as well as fixed operating costs, such as salaries, repairs, insurance, energy to operate equipment, and taxes, were not taken into account in the model because they are constant across milk composition and retentions. Seasonal variation in milk properties was also not considered. The time frame of the model was one day. Because NPN was a minimal part of whey and whey by-products, it was combined with and considered part of the whey protein component. The lactic acid content of the whey and whey by-products was included in the lactose component.

Objective Functions

The model was developed and solved in 2 parts to maximize profit from products sold. The sole purpose of the first part of the model was to determine the fat-reduced whey composition that was used in the second part of the model as a composition constraint, but not as a quantity constraint, for fat-reduced whey (Figure 1). Thus, in part 2 the quantities of products purchased, sold, or both were independent of the results from part 1. The objective function in part 1 (equation [1]) maximizes profit (PROFIT1) from producing Cheddar cheese (Cheese), from cream sold (CRS), whey cream sold (WCS), fat-reduced whey (FRW) sold (FRWS), and CN fines sold (CNF) over the cost of the inputs milk, condensed skim milk (CSM), NDM, and cream bought (CRB). Inputs and products were described on a component basis represented by I = (whey protein, ash, CN, fat, water, and lactose):

$$\begin{aligned} \text{PROFIT1} = & (\text{Cheese} \times \text{PriceCheese}) \\ & + (\text{CRS} \times \text{PriceCRS}) + (\text{WCS} \times \text{PriceWCS}) \\ & + (\text{FRW} \times \text{PriceFRWS}) + (\text{CNF} \times \text{PriceCNF}) \quad [1] \\ & - (\text{Milk} \times \text{CostMilk}) - (\text{CSM} \times \text{CostCSM}) \\ & - (\text{NDM} \times \text{CostNDM}) - (\text{CRB} \times \text{CostCRB}). \end{aligned}$$

The objective function from part 2 (equation [2]) maximized profit (PROFIT2) from overall costs and revenues for all inputs (milk, CSM, NDM, and CRB) and products, including cheese, CRS, WCS, FRWS, CNF, demineralized whey (DMW), 34% whey protein concentrate (D34), 80% whey protein concentrate (D80), lactose powder (LP), and leftover permeate sold as cow feed (CF):

$$\begin{aligned} \text{PROFIT2} = & (\text{Cheese} \times \text{PriceCheese}) \\ & + (\text{CRS} \times \text{PriceCRS}) + (\text{WCS} \times \text{PriceWCS}) \end{aligned}$$

$$\begin{aligned} & + (\text{FRWS} \times \text{PriceFRWS}) + (\text{DMW} \times \text{PriceDMW}) \\ & + (\text{D34} \times \text{PriceD34}) + (\text{D80} \times \text{PriceD80}) \quad [2] \\ & + (\text{CNF} \times \text{PriceCNF}) + (\text{LP} \times \text{PriceLactose}) \\ & + (\text{CF} \times \text{PriceCowFeed}) - (\text{Milk} \times \text{CostMilk}) \\ & - (\text{CSM} \times \text{CostCSM}) - (\text{NDM} \times \text{CostNDM}) \\ & - (\text{CRB} \times \text{CostCRB}). \end{aligned}$$

The costs of inputs and prices of products are listed in Table 1.

Constraint Equations

Equations constrained the composition of products, limited retentions of an input component into a product, and balanced inputs to sum to products (Table 2). Constraints on the composition of Cheddar cheese were based on legal limits by FDA Federal Standards of Identity (US Department of Health and Human Services, FDA, 1998) for water in cheese, and maximum CN-to-fat ratios in cheese (equations [42] to [47]). Constraints on water and fat in whey cream (equations [57] to [69]) and water, protein, ash, fat, and lactose in demineralized whey, 34% whey protein concentrate, and 80% whey protein concentrate (equations [80] to [109]) were based on minimum standard compositions of the products.

Balance equations ensured that components from an input equaled components in product(s). In the model, milk, cream, NDM, condensed skim milk, whey cream, and starter and salt components sum to components in cheese, salt whey, and whey, as found in Table 2, equations [43], [48], and [49]. Whey components sum to components in fat-reduced whey, whey cream, and CN fines (Table 2, equation [56]), and fat-reduced whey components sum to components in demineralized whey, 34% whey protein concentrate, and 80% whey protein concentrate and their respective permeates, as described in Table 2, equations [76], [80], [91], and [101]).

Compositions

All inputs and products were broken into component compositions of whey protein (mostly albumin), ash, CN, fat, water, and lactose, represented by subscript I in the equations. Standard compositions were used for condensed skim milk, NDM, cream, and starter and salt (Table 3). Milk compositions were based on data collected over 3 yr (2000 to 2002) from the University of California–Davis dairy herd. Milk CP, CN, fat, and lactose were determined by standard methods (methods 972.16 and 998.06; AOAC, 2002). For the purposes of

Table 1. Definition of terms in the objective functions

Decision variable	Definition	Value
Cheese	Cheddar cheese produced	kg
PriceCheese	Price of cheese	\$3.02/kg ¹
CRS	Cream sold	kg
PriceCRS	Price of cream	\$4.16/kg ²
WCS	Whey cream sold	kg
PriceWCS	Price of whey cream sold	\$1.74/kg ³
FRW	Fat-reduced whey sold (part 1 of the model)	kg
FRWS	Fat-reduced whey sold (part 2 of the model)	kg
PriceFRWS	Price of fat-reduced whey sold	\$0.36/kg ¹
DMW	Demineralized whey	kg
PriceDMW	Price of demineralized whey	\$1.25/kg ⁴
D34	34% whey protein concentrate	kg
PriceD34	Price of 34% whey protein concentrate	\$1.61/kg ⁵
D80	80% whey protein concentrate	kg
PriceD80	Price of 80% whey protein concentrate	\$4.74/kg ⁶
CNF	CN fines	kg
PriceCNF	Price of CN fines	\$0/kg ⁴
LP	Lactose powder	kg
PriceLactose	Price of lactose powder	\$1.74/kg ⁴
CF	Cow feed (permeate) sold	kg
PriceCowFeed	Price of cow feed (permeate) sold	\$0.024/kg ⁴
Milk	Milk	100,000 kg
CostMilk	Cost of milk	\$0.26/kg ¹
CSM	Condensed skim milk	kg
CostCSM	Cost of condensed skim milk	\$1.92/kg ²
NDM	Nonfat dry milk	kg
CostNDM	Cost of nonfat dry milk	\$1.77/kg ¹
CRB	Cream bought	kg
CostCRB	Cost of cream bought	\$4.16/kg ²

¹From CDFA (2004).²From USDA (2004).³Price based on fat content. Fat value at the price of AA butter from CDFA (1998), then the multiplier of 1.1 (A. DeRooy, Hilmar Cheese Co., Hilmar, CA; personal communication, June 23, 1998) was used to convert to the whey cream price.⁴Mullinax, D. (Hilmar Cheese Co., Hilmar, CA, personal communication, Dec. 4, 2000).⁵From USDA (2000).⁶C. Curran (USDA, personal communication, Dec. 7, 2000).

this paper, whey protein was determined as milk CP minus CN. Ash was determined from data (Zadow, 1992) that included composition changes over 12 mo. Whey protein, CN, fat, and lactose were varied individually based on high and low values from the University of California–Davis data set for 11 different milk compositions, as listed in Table 4.

Retentions

Minimum and maximum limits on retentions constrained the amount of component from an input that could appear in a product. Retention constraints were used for milk components to cheese and whey, whey components to fat-reduced whey and whey cream, and permeate components to lactose powder. Values for retention minimum and maximum equations milk to whey, whey to whey cream, whey to fat-reduced whey, and permeate to lactose powder retentions were calculated from estimating the proportion of each component

that ended up in product using standard product compositions (Table 5). Retentions of milk components to cheese were based on data from Schaar (1985) and Walsh et al. (1995) on changes in protein and fat retentions attributable to either the κ -CN AA genotype or the κ -CN BB genotype (Table 6). Because protein retention of milk to cheese in the model was composed of a whey protein fraction and a CN fraction, the whey protein fraction of the retention was assumed fixed at 3.4% and the CN fraction of the retention made up the difference. Ten different sets of milk to cheese retentions were used (Table 7).

Sensitivity Analysis and Statistical Analysis

To assess the effects of different milk compositions and retentions on cheese and whey production and profit, 110 simulations were run, including all combinations of milk compositions and cheese retentions. Eleven different milk compositions (Table 4) and 10

Table 2. Constraint equations of the Cheddar cheese plant model¹

Equation description	Equation	Equation no.
Part 1—Cheese production		
Milk		
Amount of milk purchased (MILK)	$MILK < 100,000$	3
Amount of components in milk (MILK _I)	$MILK_I^2 = MILK \times PercentMILK_I$	4
Amount of components in skim milk (SMILK _I) after subtracting cream components from milk (CRM _I)	$SMILK_I = MILK_I - CRM_I$	5
Minimum components from SMILK to cheese ³	$MCheese_I > MinCheese_I \times SMILK_I$	6
Maximum components from SMILK to cheese ³	$MCheese_I < MaxCheese_I \times SMILK_I$	7
Minimum components from SMILK to whey ³	$MWhey_I > MinWhey_I \times SMILK_I$	8
Maximum components from SMILK to whey ³	$MWhey_I < MaxWhey_I \times SMILK_I$	9
SMILK (M) components to cheese, whey, and salt whey (SW _I)	$SMILK_I = MCheese_I + MWhey_I + MSW_I$	10
Cream (CR)		
CR bought (CRB)	$CRB = \sum_I CRB_I$	11
CR sold (CRS)	$CRS = \sum_I CRS_I$	12
Amount of components in CR from milk (CRM _I)	$CRM_I = (\sum_I CRM_I) \times PercentCR_I$	13
Amount of components in CRB (CRB _I)	$CRB_I = CRB \times PercentCR_I$	14
Amount of components in CRS (CRS _I)	$CRS_I = CRS \times PercentCR_I$	15
Minimum components from CR to cheese ³	$CRCheese_I > MinCheese_I \times CR_I$	16
Maximum components from CR to cheese ³	$CRCheese_I < MaxCheese_I \times CR_I$	17
Minimum components from CR to whey ³	$CRWhey_I > MinWhey_I \times CR_I$	18
Maximum components from CR to whey ³	$CRWhey_I < MaxWhey_I \times CR_I$	19
CR components from milk (CRM) and CRB minus CRS	$CR_I = CRM_I + CRB_I - CRS_I$	20
Cream components into cheese, whey, and SW	$CR_I = CRCheese_I + CRWhey_I + CRSW_I$	21
NDM		
NDM purchased (NDM)	$NDM = \sum_I NDM_I$	22
Amount of components in NDM (NDM _I) purchased	$NDM_I = NDM \times PercentNDM_I$	23
Minimum components from NDM to cheese ³	$NDM_Cheese_I > MinCheese_I \times NDM_I$	24
Maximum components from NDM to cheese ³	$NDM_Cheese_I < MaxCheese_I \times NDM_I$	25
Minimum components from NDM to whey ³	$NDMWhey_I > MinWhey_I \times NDM_I$	26
Maximum components from NDM to whey ³	$NDMWhey_I < MaxWhey_I \times NDM_I$	27
NDM components into cheese, whey, and SW	$NDM_I = NDM_Cheese_I + NDMWhey_I + NDMSW_I$	28
Condensed skim milk (CSM)		
CSM purchased (CSM)	$CSM = \sum_I CSM_I$	29
Amount of components in CSM (CSM _I)	$CSM_I = CSM \times PercentCSM_I$	30
Minimum components from CSM to cheese ³	$CSM_Cheese_I > MinCheese_I \times CSM_I$	31
Maximum components from CSM to cheese ³	$CSM_Cheese_I < MaxCheese_I \times CSM_I$	32
Minimum components from CSM to whey ³	$CSMWhey_I > MinWhey_I \times CSM_I$	33
Maximum components from CSM to whey ³	$CSMWhey_I < MaxWhey_I \times CSM_I$	34
CSM components into cheese, whey, and SW	$CSM_I = CSM_Cheese_I + CSMWhey_I + CSMSW_I$	35
Starter and salt (SS)		
Amount of components in SS (SS _I) is a fixed proportion of total inputs [SMILK, CR, NDM, CSM, WC (whey cream)]	$SS_I = \sum_I (SMILK_I + CR_I + NDM_I + CSM_I + WC_I) \times PercentSS_I$	36
Minimum components from SS to cheese ³	$SS_Cheese_I > MinCheese_I \times SS_I$	37
Maximum components from SS to cheese ³	$SS_Cheese_I < MaxCheese_I \times SS_I$	38
Minimum components from SS to whey ³	$SSWhey_I > MinWhey_I \times SS_I$	39
Maximum components from SS to whey ³	$SSWhey_I < MaxWhey_I \times SS_I$	40
SS components into cheese, whey, and SW	$SS_I = SSCheese_I + SSWhey_I + SSSW_I$	41
Cheese		
Cheese sold (Cheese)	$Cheese = \sum_I Cheese_I$	42
Amount of components in cheese is the sum of input components based on retentions for each input component	$Cheese_I = (MCheese_I + CRCheese_I + NDM_Cheese_I + CSM_Cheese_I + SSCheese_I + WC_Cheese_I)$	43
Water less than 38% of cheese	$Cheese_{Water} < 0.38 \times \sum_I Cheese_I$	44
Fat in cheese greater than 50% (dry basis)	$Cheese_{Fat} > 0.5 \times (Cheese - Cheese_{Water})$	45
CN-to-fat ratio in cheese greater than 64%	$Cheese_{CN} > 0.64 \times Cheese_{Fat}$	46
CN-to-fat ratio in cheese less than 80%	$Cheese_{CN} < 0.80 \times Cheese_{Fat}$	47
Salt whey (SW)		
Amount of components in SW (SW _I) is the sum of input component compositions based on retentions	$SW_I = MSW_I + CRSW_I + NDMSW_I + CSMSW_I + SSSW_I + WCSW_I$	48
Whey		
Amount of components in whey is the sum of input component compositions based on retentions	$Whey_I = MWhey_I + CRWhey_I + NDMWhey_I + CSMWhey_I + SSWhey_I + WCWhey_I$	49
Minimum components from whey to fat-reduced whey (FRW) ³	$Whey_{FRW_I} > MinFRW_I \times Whey_I$	50
Maximum components from whey to FRW ³	$Whey_{FRW_I} < MaxFRW_I \times Whey_I$	51

Continued

Table 2 (Continued). Constraint equations of the Cheddar cheese plant model¹

Equation description	Equation	Equation no.
Minimum components from whey to WC ³	$WheyWC_I > MinWC_I \times Whey_I$	52
Maximum components from whey to WC ³	$WheyWC_I < MaxWC_I \times Whey_I$	53
FRW sold (FRW) is from whey retention equations	$FRW = \sum_I WheyFRW_I$	54
CN fines sold (CNF) is from whey retention equations	$CNF = \sum_I WheyCNF_I$	55
Whey components output into FRW, WC, and CNF	$Whey_I = WheyFRW_I + WheyWC_I + WheyCNF_I$	56
Whey cream (WC)		
WC (WC _I) component composition is from whey retention equations	$WC_I = WheyWC_I$	57
WC components not sold (WCC _I); WC components sold (WCS _I)	$WCC_I = WC_I - WCS_I$	58
Minimum components from WC to cheese ³	$WC Cheese_I > MinCheese_I \times WCC_I$	59
Maximum components from WC to cheese ³	$WC Cheese_I < MaxCheese_I \times WCC_I$	60
Minimum components from WC to whey ³	$WC Whey_I > MinWhey_I \times WCC_I$	61
Maximum components from WC to whey ³	$WC Whey_I < MaxWhey_I \times WCC_I$	62
WCC components into cheese, whey, and SW	$WCC_I = WCCheese_I + WCWhey_I + WCSW_I$	63
WC sold (WCS)	$WCS = \sum_I WCS_I$	64
Component fat in WC fixed at 34%	$WC_{Fat} = 0.34 \times \sum_I WC_I$	65
Component fat in WCC fixed at 34%	$WCC_{Fat} = 0.34 \times \sum_I WCC_I$	66
Component fat in WCS fixed at 34%	$WCS_{Fat} = 0.34 \times \sum_I WCS_I$	67
WCC fat from milk less than 5% of fat in cheese	$WCCheese_{Fat} < 0.05 \times Cheese_{Fat}$	68
Water from WCC equals 49.5% of water in SW	$WCSW_{Water} = 0.495 \times WCC_{Water}$	69
Part 2—Whey protein product production		
Fat-reduced whey (FRW)		
Percent FRW _I is fixed from solution in part 1 retention equations of whey components going to FRW	$PercentFRW_I = WhFRW_I / (\sum_I FRW_I)$	70
FRW sold (FRWS)	$FRWS = \sum_I FRWS_I$	71
FRW components not sold (FRWC _I) equals total FRW minus FRW sold (FRWS _I)	$FRWC_I = FRW_I - FRWS_I$	72
Component composition of FRW (total)	$FRW_I = (\sum_I FRW_I) \times PercentFRW_I$	73
Component composition of FRWC (not sold)	$FRWC_I = (\sum_I FRWC_I) \times PercentFRW_I$	74
Component composition of FRWS (sold)	$FRWS_I = (\sum_I FRWS_I) \times PercentFRW_I$	75
Total FRWC equals FRW going to DMW, D34 and D80	$FRWC_I = FRWDMW_I + FRWD34_I + FRWD80_I$	76
FRWC going to DMW has the same composition as FRW set from part 1	$FRWDMW_I = PercentFRW_I \times (\sum_I FRWDMW_I)$	77
FRWC going to D34 has the same composition as FRW set from part 1	$FRWD34_I = PercentFRW_I \times (\sum_I FRWD34_I)$	78
FRWC going to D80 has the same composition as FRW set from part 1	$FRWD80_I = PercentFRW_I \times (\sum_I FRWD80_I)$	79
Demineralized whey (DMW)		
FRWC to DMW equals DMW produced plus permeate (leftover)	$FRWDMW_I = DMW_I + PermeateDMW_I$	80
DMW sold (DMW)	$DMW = \sum_I DMW_I$	81
Less than 6% water in DMW	$DMW_{Water} < 0.06 \times (\sum_I DMW_I)$	82
Whey protein (WP) and CN in DMW greater than 15%	$DMW_{WP} + DMW_{CN} > 0.15 \times (\sum_I DMW_I)$	83
WP and CN in DMW less than 24%	$DMW_{WP} + DMW_{CN} < 0.24 \times (\sum_I DMW_I)$	84
Ash in DMW greater than 1%	$DMW_{Ash} > 0.01 \times (\sum_I DMW_I)$	85
Ash in DMW less than 7%	$DMW_{Ash} < 0.07 \times (\sum_I DMW_I)$	86
Fat in DMW greater than 1%	$DMW_{Fat} > 0.01 \times (\sum_I DMW_I)$	87
Fat in DMW less than 4%	$DMW_{Fat} < 0.04 \times (\sum_I DMW_I)$	88
Lactose in DMW greater than 70%	$DMW_{Lactose} > 0.70 \times (\sum_I DMW_I)$	89
Lactose in DMW less than 85%	$DMW_{Lactose} < 0.85 \times (\sum_I DMW_I)$	90
Whey with 34% protein (D34)		
FRWC to 34% protein whey (D34) equals D34 produced plus permeate (leftover)	$FRWD34_I = D34_I + PermeateD34_I$	91
Whey protein concentrate at 34% protein sold	$D34 = \sum_I D34_I$	92
Less than 4% water in D34	$D34_{Water} < 0.04 \times (\sum_I D34_I)$	93
Whey protein and CN in D34 greater than 34%	$D34_{WP} + D34_{CN} > 0.34 \times (\sum_I D34_I)$	94
Ash in D34 greater than 7%	$D34_{Ash} > 0.07 \times (\sum_I D34_I)$	95
Ash in D34 less than 9%	$D34_{Ash} < 0.09 \times (\sum_I D34_I)$	96
Fat in D34 greater than 1%	$D34_{Fat} > 0.01 \times (\sum_I D34_I)$	97
Fat in D34 less than 5%	$D34_{Fat} < 0.05 \times (\sum_I D34_I)$	98
Lactose in D34 greater than 40%	$D34_{Lactose} > 0.40 \times (\sum_I D34_I)$	99
Lactose in D34 less than 55%	$D34_{Lactose} < 0.55 \times (\sum_I D34_I)$	100

Continued

Table 2 (Continued). Constraint equations of the Cheddar cheese plant model¹

Equation description	Equation	Equation no.
Whey with 80% protein (D80)		
FRWC to 80% protein whey (D80) equals D80 produced plus permeate (leftover)	$FRWD80_I = D80_I + \text{Permeate}D80_I$	101
Whey protein concentrate at 80% protein sold (D80)	$D80 = \sum_I D80_I$	102
Less than 4% water in D80	$D80_{\text{Water}} < 0.04 \times (\sum_I D80_I)$	103
Whey protein (WP) and CN in D80 greater than 80%	$D80_{WP} + D80_{CN} > 0.80 \times (\sum_I D80_I)$	104
Ash in D80 greater than 3%	$D80_{\text{Ash}} > 0.03 \times (\sum_I D80_I)$	105
Ash in D80 less than 9%	$D80_{\text{Ash}} < 0.09 \times (\sum_I D80_I)$	106
Fat in D80 greater than 7%	$D80_{\text{Fat}} < 0.07 \times (\sum_I D80_I)$	107
Lactose in D80 greater than 5%	$D80_{\text{Lactose}} > 0.05 \times (\sum_I D80_I)$	108
Lactose in D80 less than 30%	$D80_{\text{Lactose}} < 0.30 \times (\sum_I D80_I)$	109
Lactose powder (LP)		
Total permeate equals permeate from DMW, D34, and D80	$\text{Permeate}_I = \text{Permeate}DMW_I + \text{Permeate}D34_I + \text{Permeate}D80_I$	110
Minimum components from permeate to LP ³	$\text{Permeate}LP_I > \text{Min}LP_I \times \text{Permeate}_I$	111
Maximum components from permeate to LP ³	$\text{Permeate}LP_I < \text{Max}LP_I \times \text{Permeate}_I$	112
LP sold is from permeate retentions and is limited by the equation below	$LP = \sum_I \text{Permeate}LP_I$	113
Water in LP less than 5%	$\text{Permeate}LP_{\text{Water}} < 0.05 \times (\sum_I LP_I)$	114
Cow feed (CF)		
Permeate to CF equals total permeate minus permeate from LP	$\text{Permeate}CF_I = \text{Permeate}_I - \text{Permeate}LP_I$	115
CF sold (CF)	$CF = \sum_I CF_I$	116
CF (CF _I) component composition is SW components plus permeate components from LP	$CF_I = SW_I + \text{Permeate}CF_I$	117

¹Units are in kilograms unless otherwise noted.

²Index I represents the composition of inputs and products as (whey protein, ash, CN, fat, water, and lactose); therefore, each equation indexed by I represents 6 equations, one for each component.

³Retention equations limiting the minimum and maximum components (I) transferred to each product.

sets of retentions on cheese (Table 7) were assumed to represent milk of different genotypes because a particular milk genotype might contribute more to profit than another genotype, depending on which products a cheese plant produces. The retentions described in Table 5 were fixed for all simulations.

Separate runs were done for each simulation if NDM or cream bought were not purchased or if demineralized whey, 34% whey protein concentrate, or 80% whey protein concentrate were not produced. The cost of NDM or cream bought and the prices for demineralized whey,

34% whey protein concentrate, or 80% whey protein concentrate were adjusted to determine the cost or price necessary for these products to enter the solution (Tables 10 to 14). Marginal values output by GAMS were unable to be used because there was too much interdependence between the variables. Runs for values of cream bought included the assumption that cream sold was zero (CRS = 0). Results of simulations for different milk compositions and milk-to-cheese retentions based on genotype were compared using the statistical model

$$Y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij},$$

where α_i is the milk composition and $i = 1, 2, \dots, 6$ (Table 4), and β_j is the genotype and $j = 1, 2, \dots, 8$ (Table 7) using PROC GLM of SAS v. 8.1 (SAS Institute, 2002). Multiple means were compared within the main effects milk composition and genotype using least squares means comparisons.

RESULTS

The major products produced and the associated profit from the simulation results are presented in Tables 8 and 9 for different milk compositions and retentions (genotypes), respectively. The percent composi-

Table 3. Standard composition of input ingredients

Component	CSM, ¹ %	NDM, ² %	CR, ³ %	SS, ⁴ %
WP ⁵	1.67	6.57	0.54	5.00
Ash	2.30	7.98	0.58	1.70
CN	8.13	29.47	2.14	0.00
Fat	0.30	0.90	30.08	0.00
Water	73.00	3.43	62.98	0.00
Lactose	14.60	51.65	3.68	0.00

¹Condensed skim milk, from Wong (1988).

²Nonfat dry milk, averaged from Chandan (1997) and Wong (1988).

³Cream, from Chandan (1997).

⁴Starter and salt, D. Mullinax (Hilmar Cheese Company, Hilmar, CA, personal communication, Dec. 4, 2000).

⁵WP = Whey protein.

Table 4. Milk compositions based on data from the University of California at Davis dairy herd

Item	Average, ¹ 3.25% protein	3.25% protein ² (low WP), %	3.25% protein ³ (high WP), %	Low CN, ⁴ %	High CN, ⁴ %	Low WP, ⁵ %	High WP, ⁵ %	Low fat, ⁶ %	High fat, ⁶ %	Low lactose, ⁷ %	High lactose, ⁷ %
WP	0.74	0.41	1.14	0.74	0.74	0.41	1.14	0.74	0.74	0.74	0.74
Ash	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70
CN	2.51	2.84	2.11	1.62	3.59	2.51	2.51	2.51	2.51	2.51	2.51
Fat	3.98	3.98	3.98	3.98	3.98	3.98	3.98	1.89	5.97	3.98	3.98
Water	87.00	87.00	87.00	87.89	85.92	87.33	86.60	89.09	85.01	88.01	86.43
Lactose	5.07	5.07	5.07	5.07	5.07	5.07	5.07	5.07	5.07	4.06	5.64

¹Average composition from 1,528 cow records over 3 yr (2000 to 2002) ranging from 9 to 700 DIM.

²Lowest percentage of whey protein (WP) found in records. The percentage of CN was adjusted to 3.25% of total protein.

³Highest percentage of WP in found in records. The percentage of CN was adjusted to 3.25% of total protein.

⁴Highest and lowest percentages of CN from cow records. The percentage of WP was set at the average, so total protein varies.

⁵Highest and lowest percentages of WP from cow records. The percentage of CN was set at the average, so total protein varies.

⁶Highest and lowest percentages of fat from cow records. The percentage of water changed, so the percentage composition sums to 100.

⁷Highest and lowest percentages of lactose from cow records. The percentage of water changed, so the percentage composition sums to 100.

Table 5. Retention ranges for milk and whey components to processed products (units in kg of component/kg of input)

Component	Milk to whey ¹		Whey to WC ²		Whey to FRW ³		Permeate to LP ⁴	
	MinWhey _I	MaxWhey _I	MinWC _I	MaxWC _I	MinFRW _I	MaxFRW _I	MinLP _I	MaxLP _I
WP ⁵	0.51	0.51	0.0	0.0	0.90	0.90	0.13	0.15
Ash	0.11	0.11	0.0	0.0	0.98	0.98	0.018	0.018
CN	0.004	0.004	0.0	0.0	0.70	0.70	0.90	0.90
Fat	0.04	0.04	0.5	0.75	0.25	0.25	0.11	0.11
Water	0.0	0.95	0.002	0.005	0.80	0.995	0.00028	0.00028
Lactose	0.45	0.45	0.0	0.0	0.99	0.99	0.99	0.99

¹MinWhey_I and MaxWhey_I are the minimum and maximum retentions for the Ith component for milk to whey and are found in Table 2 in equations [8], [18], [26], [33], [39], and [9], [19], [27], [34], and [40], respectively.

²MinWC_I and MaxWC_I are the minimum and maximum retentions for the Ith component for whey to whey cream (WC) and are found in Table 2 in equations [52] and [53], respectively.

³MinFRW_I and MaxFRW_I are the minimum and maximum retentions for the Ith component for whey to fat-reduced whey (FRW) and are found in Table 2 in equations [50] and [51], respectively.

⁴MinLP_I and MaxLP_I are the minimum and maximum retentions for the Ith component for permeate to lactose powder (LP) and are found in Table 2 in equations [111] and [112], respectively.

⁵WP = Whey protein.

Table 6. Changes in retention of milk to cheese attributable to different κ -CN milk protein genotypes (AA or BB) from the literature¹

Component	AA1 ²	BB1 ²	Average 1 ³	AA2 ⁴	BB2 ⁴	Average 2 ⁵
Protein	0.749	0.756	0.7525	0.7601	0.7751	0.7676
Fat	0.915	0.921	0.9180	0.7795	0.8993	0.8394

¹Units in kg of component/kg of input.

²From Schaar (1985).

³Average of protein and fat retentions of AA1 and BB1 genotype values, from Schaar (1985).

⁴Walsh et al. (1995).

⁵Average of protein and fat retentions of AA2 and BB2 genotype values, from Walsh et al. (1995).

Table 7. Fixed retentions of CN and fat to cheese attributable to different κ -CN milk protein genotypes used in model simulations¹

Genotype ²	CN retentions	Fat retentions
AA protein (AA1P)	0.7150	0.9180
BB protein (BB1P)	0.7220	0.9180
Average 1 (AVG1)	0.7185	0.9180
AA protein (AA2P)	0.7261	0.8394
BB protein (BB2P)	0.7411	0.8394
Average 2 (AVG2)	0.7336	0.8394
AA fat (AA1F)	0.7185	0.9150
BB fat (BB1F)	0.7185	0.9210
AA fat (AA2F)	0.7336	0.7795
BB fat (BB2F)	0.7336	0.8993

¹Units in kg of component/kg of input using average values from Table 6 corrected for whey protein retention. Whey protein and ash retentions were fixed at 0.034 and 0.16, respectively, for all genotypes. Water retentions were a minimum of 0 and a maximum of 1, and lactose retentions were a minimum of 0.045 and a maximum of 0.05 for all genotypes.

²AA and BB are κ -CN genotypes, 1 (e.g., AA1P) is based on protein and fat retentions from Schaar (1985), and 2 (e.g., AA2P) is based on protein and fat retentions from Walsh et al. (1995). For CN retentions: P (e.g., AA1P), AVE1 and AVE2 indicate the numbers are from Table 6 protein values corrected for whey protein and F (e.g., AA1F) indicates the numbers are averages of protein values AA1 and BB1 or AA2 and BB2 from Table 6 protein values corrected for whey protein. For fat retentions: P, AVE1, and AVE2 indicate the numbers are from averages of fat values AA1 and BB1 or AA2 and BB2 from Table 6 fat values and F indicates the numbers are from Table 6 fat values. Numbers correspond to CN and fat retentions for MinCheese₁ and MaxCheese₁ as described in Table 2 in equations [6], [16], [24], [31], and [37], and in equations [7], [17], [25], [32], and [38], respectively.

tion of cheese, fat-reduced whey, and cream were not significantly different among the simulations and are not reported. A high-CN milk composition (4.33% protein, 3.98% fat) resulted in significantly higher cheese yield, followed by a low-fat milk composition (3.25% protein, 1.89% fat), which was not significant. A high-fat milk composition (3.25% protein, 5.97% fat) produced the lowest amount of cheese and the most cream

sold. In simulations with a low-CN milk composition and low-whey protein milk composition including all genotypes, NDM was bought to increase CN input into the plant (2,135 and 444 kg of NDM bought, respectively).

Cream was bought in all simulations with low-fat milk compositions (3,350 kg of cream bought) and high-CN milk compositions with genotype AA2F (CN retention of 0.7336 and fat retention of 0.7795) (422 kg of cream bought) to increase the fat input into the plant. Cream was bought with the AA2F genotype and high-CN milk composition because retentions were comparatively higher on CN and lower on fat. In simulations in which cream was bought, the fat-reduced whey sold was highest and cream was not sold. As a result, simulations in which cream was bought (low-cream milk compositions) resulted in the lowest profit. The high-CN milk composition resulted in significantly higher cheese and less cream sold (Table 8).

The high-fat milk composition produced the lowest amount of cheese and fat-reduced whey sold because fat was diverted to cream sold, resulting in the highest profit from any of the milk composition simulations (Table 8). Cream sold, fat-reduced whey sold, and cheese produced were dependent on the fat and CN input into the plant. However, overall profit was dependent on the fat content and fat retention in cheese because of the high price of cream. In general, the more cheese that was produced, the less cream was sold (Table 8).

Milk composition affected cheese production and profit more than did different retentions (genotypes; Tables 8 and 9). Changes in composition (Table 4) resulted in a 3,000-kg difference in cheese production and a \$54,000 difference in profit (Table 8). Changes in retentions (Table 7) resulted in a 970-kg difference in cheese production and a \$5,878 difference in profit

Table 8. Inputs and products produced as predicted by model simulations of cheese making with different milk compositions¹

Milk composition	Cheese, kg	FRWS, kg	CRS, kg	Profit, \$
Average 3.25% protein	7,442	82,382	4,329	44,274
Low WP, 3.25% protein	7,940	82,460	3,953	44,237
High WP, 3.25% protein	7,197	82,683	4,370	43,039
Low CN	7,344	83,877	4,318	40,710
High CN	9,846*	83,070*	2,223*	40,464
Low WP	7,440	82,556	4,295	44,187
High WP	7,444	82,171	4,370	44,378
Low fat	8,004	89,193*	0*	16,490*
High fat	6,834*	74,480*	11,781*	70,586*
Low lactose	7,471	84,052	4,113	44,058
High lactose	7,462	82,131	4,380	44,462
SEM	91	79	215	257

¹As described in Table 4. FRWS = fat-reduced whey sold; CRS = cream sold; WP = whey protein.

*Different from all other products produced or profits within a column ($P < 0.006$).

Table 9. Inputs and products produced as predicted by model simulations of cheese making with different milk to cheese retentions¹

Retention	Cheese, kg	FRWS, kg	CRS, kg	Profit, \$
AA1P	7,720	82,600	4,421	43,763
BB1P	7,748	82,603	4,985	43,827
AVG1	7,734	82,602	4,415	43,795
AA2P	7,435	82,679	4,226	42,380
BB2P	7,509	82,703	4,179	42,444
AVG2	7,470	82,690	4,204	42,416
AA1F	7,720	82,603	4,411	43,747
BB1F	7,747	82,601	4,419	43,843
AA2F	7,329	82,910	3,886	40,602*
BB2F	7,716	82,624	4,352	43,587
SEM	91	79	215	257

¹Symbols represent different genotype combinations of CN and fat retentions from Table 7 used in simulations. FRWS = fat-reduced whey sold; CRS = cream sold.

*Different from all other profits ($P < 0.0001$).

(Table 9). The BB1P (CN retention of 0.7220 and fat retention of 0.9180) retention resulted in the most cheese produced and the most cream sold. The lowest amount of cheese produced, the most fat-reduced whey sold, the lowest cream sold, and the lowest profit were from the AA2F retention. Therefore, given the prices and compositions used in the simulations, the best genotypes for cheese production and profit were BB1F (CN retention of 0.7185 and fat retention of 0.9210) and BB1P. The BB1F profit was \$48 over AVG1 (CN retention of 0.7185 and fat retention of 0.9180) for 13 kg more cheese and the BB1P profit was \$32 over AVG1 for 14 kg more cheese.

Different milk composition–genotype combinations also resulted in different maximum prices that a processing plant would be willing to pay for NDM (as a CN source) and cream bought (as a fat source) to optimize profit from cheese and whey production. For the high-CN composition, the value of NDM was \$0/kg across all genotypes (Table 10). However, for the low-CN composition, the value of NDM averaged \$2.13/kg across genotypes. For the AA2F genotype, the value of NDM was also \$0/kg except for the low-CN and low-whey protein (3.25% protein) compositions. The overall average, excluding the high-CN composition row and the AA2F genotype column, was \$1.50/kg. The overall average value of cream bought was \$1.38/kg (Table 11), excluding the low-fat composition row (average \$11.61/kg) and the high-CN milk composition–AA2F low-fat-retention genotype (\$83.73/kg). The combination of high-CN milk composition and low-fat-retention genotype (AA2F) greatly increased the value of cream to the cheese plant.

Because of the high value of cream, the dried whey products (demineralized whey, 34% whey protein concentrate, and 80% whey protein concentrate) were not produced in the sensitivity analysis runs. However,

when the selling prices were adjusted so that they would be produced, the price depended on the quantity of protein in the dried whey product and the quantity of protein in the milk. Tables 12 to 14 show the minimum selling prices for demineralized whey, 34% whey protein concentrate, and 80% whey protein concentrate, respectively. Eighty percent whey protein concentrate (at \$57.62/kg) was worth more, on average, followed by 34% whey protein concentrate (at \$26.66/kg) and then demineralized whey (\$12.50/kg). The high-whey protein, 3.25% total protein, and low-whey protein milk compositions, 2.92% total protein (Table 4), resulted in the highest average values for these 2 rows across genotypes for demineralized whey (\$19.68/kg), 34% whey protein concentrate (\$42.56/kg), and 80% whey protein concentrate (\$98.14/kg). The low-CN composition, 2.36% total protein, resulted in the lowest average values across genotypes for demineralized whey (\$9.58/kg) and 34% whey protein concentrate (\$19.93/kg), and low-whey protein, 3.25% total protein composition, resulted in the lowest average value across genotypes for 80% whey protein concentrate (\$34.05/kg). Genotype (retentions) had a small effect on the value of dried whey products within a given milk composition.

DISCUSSION

Based on results from the sensitivity analyses and determination of input and output values, milk composition had a much greater effect on cheese yield than did retention effects attributable to different κ -CN genotypes. Few data sets are available on the effects of κ -CN genotypes on retentions and milk composition. Data from only 2 studies were found and used to set protein and fat retentions in the model (Table 6). In both data sets, the AA genotype had consistently lower retention for protein and fat than did the BB genotype. Data from Walsh et al. (1995) showed a greater difference between AA and BB retentions than did data from Schaar (1985). As a result, retentions from Walsh et al. (1995) resulted in more cheese production from the BB genotype, compared with the AA genotype. Based on these data, the BB genotype protein retention effects resulted in 51 kg more cheese than did the AA genotype protein retentions, and the BB genotype fat-retention effects resulted in 207 kg more cheese than did the AA genotype fat retentions (Table 9). The κ -CN genotype may also affect milk composition; however, the data available were insufficient to quantify the effect when trimester and season of milk production were also considered. Sapru et al. (1997) examined the effects of stage of lactation and milking frequency on milk composition and Cheddar cheese yield. Using data from 100 cows, they found that the percentage of fat increased (3.50 to

Table 10. Maximum prices (\$/kg) willing to pay for NDM to use in cheese processing for each milk composition¹ and genotype combination²

Milk composition	Genotype										Average ³
	AA1P	BB1P	AVG1	AA2P	BB2P	AVG2	AA1F	BB1F	AA2F	BB2F	
Average 3.25% protein	1.44	1.45	1.45	1.46	1.48	1.45	1.45	1.45	0	1.47	1.31
Low WP 3.25% protein	2.15	2.16	2.15	2.08	1.48	1.47	2.15	2.16	1.47	2.17	1.94
High WP 3.25% protein	1.44	1.45	1.45	0	0	0	1.45	1.45	0	0	0.72
Low CN	2.14	2.17	2.15	2.08	2.12	2.10	2.15	2.16	2.01	2.17	2.13
High CN	0	0	0	0	0	0	0	0	0	0	0.00
Low WP	1.44	1.45	1.45	1.46	1.48	1.47	1.45	1.45	0	1.47	1.31
High WP	1.46	1.47	1.46	1.47	1.49	1.48	1.46	1.46	0	1.48	1.32
Low fat	1.44	1.45	1.45	1.46	1.48	1.47	1.45	1.45	0	1.47	1.31
High fat	1.44	1.45	1.45	1.46	0	1.47	1.45	1.45	0	1.47	1.16
Low lactose	1.44	1.45	1.45	1.46	1.48	1.47	1.45	1.45	0	1.47	1.31
High lactose	1.44	1.45	1.45	1.46	1.48	1.47	1.45	1.45	0	1.47	1.31

¹See Table 4.²Symbols represent different genotype combinations of CN and fat retentions from Table 7 used in simulations. WP = Whey protein.³Average across columns.

3.70) and the percentage of CN increased (2.30 to 2.37) from early to late lactation. However, the percentages of fat and CN decreased from twice-a-day milking to thrice-a-day milking (3.58 to 3.50, 2.40 to 2.36, respectively). Correspondingly, Cheddar cheese yield increased from early to late lactation by 0.59% and decreased by 0.20% with more frequent milking. However, none of the Cheddar cheese yield values were significantly different. Milk composition data used in the model were based on milk production records from the University of California–Davis dairy herd collected from 2000 to 2002. Although the herd was genotyped for κ -CN, the time (3 yr) was insufficient to detect a significant difference in milk composition because of selection for κ -CN AA or BB in milk. The effect on cheese production of using milk from cows selected for the BB κ -CN genotype could be tested more completely if more data were available on milk composition and cheese retentions with different milk protein genotypes.

The model showed that both the protein and fat contents of milk should be considered when selecting for the κ -CN BB genotype. Previous studies (Schaar, 1985; Walsh et al., 1995) showed that the κ -CN genotype affected both protein and fat retentions, which will affect cheese yield. The model quantified how differences in milk composition significantly affected cheese and whey yields (Table 9). Although CN was more important to cheese yield, both fat and CN contents determined the yield, as illustrated by the Van Slyke formula (Lucey and Kelly, 1994; Covington, 1998). Casein was important to cheese yield because it influenced rennet coagulation by way of forming the structural matrix of cheese, which influenced the retention of fat and moisture (Lucey and Kelly, 1994).

Estimated maximum prices for NDM and cream bought (Tables 10 and 11), minimum prices for dried whey products (demineralized whey, 34% whey protein concentrate, and 80% whey protein concentrate in Ta-

Table 11. Maximum prices (\$/kg) willing to pay for cream bought to use in a cheese-processing plant for each milk composition¹ and genotype combination²

Milk composition	Genotype										Average ³
	AA1P	BB1P	AVG1	AA2P	BB2P	AVG2	AA1F	BB1F	AA2F	BB2F	
Average 3.25% protein	1.39	1.40	1.40	1.32	1.34	1.33	1.39	1.40	1.26	1.40	1.36
Low WP 3.25% protein	1.39	1.40	1.40	1.32	1.34	1.33	1.39	1.40	1.26	1.40	1.36
High WP 3.25% protein	1.39	1.40	1.40	1.32	1.34	1.33	1.39	1.40	1.50	1.40	1.39
Low CN	1.39	1.40	1.40	1.32	1.34	1.33	1.39	1.40	1.26	1.40	1.36
High CN	1.70	1.70	1.70	1.58	1.59	1.58	1.69	1.70	83.73	1.67	9.86
Low WP	1.39	1.40	1.40	1.33	1.34	1.33	1.39	1.40	1.26	1.40	1.36
High WP	1.40	1.41	1.41	1.34	1.35	1.34	1.40	1.41	1.27	1.41	1.37
Low fat	13.53	13.17	13.37	9.80	9.65	9.80	13.19	13.49	8.24	11.82	11.61
High fat	1.29	1.30	1.30	1.24	1.25	1.24	1.29	1.30	1.18	1.30	1.27
Low lactose	1.39	1.40	1.40	1.33	1.34	1.33	1.39	1.40	1.26	1.40	1.36
High lactose	1.39	1.40	1.40	1.33	1.34	1.33	1.39	1.40	1.26	1.40	1.36

¹See Table 4.²Symbols represent different genotype combinations of CN and fat retentions from Table 7 used in simulations. WP = Whey protein.³Average across columns.

Table 12. Minimum prices (\$/kg) willing to sell demineralized whey produced from a cheese-processing plant for each milk composition¹ and genotype combination²

Milk composition	Genotype										Average ³
	AA1P	BB1P	AVG1	AA2P	BB2P	AVG2	AA1F	BB1F	AA2F	BB2F	
Average 3.25% protein	11.48	11.48	11.48	10.98	11.48	11.48	11.48	11.48	11.47	11.48	11.43
Low WP 3.25% protein	9.83	9.84	9.84	9.93	9.94	9.94	9.84	9.82	9.94	9.87	9.88
High WP 3.25% protein	19.63	19.63	19.63	19.55	19.53	19.54	19.61	19.64	19.45	19.62	19.58
Low CN	9.70	9.71	6.99	9.83	10.10	10.05	9.70	9.70	10.28	9.73	9.58
High CN	11.26	11.26	11.26	11.26	11.26	11.26	11.26	11.26	11.26	11.26	11.26
Low WP	19.79	19.77	19.80	19.79	19.79	19.79	19.77	19.80	19.76	19.77	19.78
High WP	9.94	9.93	9.94	9.92	9.94	9.94	9.94	9.93	9.63	9.89	9.90
Low fat	11.76	11.76	11.76	11.76	11.76	11.76	11.76	11.76	11.76	11.76	11.76
High fat	10.98	10.96	10.99	10.97	10.99	10.99	10.99	10.98	10.99	10.99	10.98
Low lactose	12.12	12.12	12.12	12.11	12.11	12.11	12.11	12.11	12.11	12.11	12.11
High lactose	11.26	11.26	11.26	11.26	11.25	11.26	11.26	11.26	11.26	11.26	11.26

¹See Table 4.²Symbols represent different genotype combinations of CN and fat retentions from Table 7 used in simulations. WP = Whey protein.³Average across columns.

bles 12 to 14), and the dollar contributions of milk composition and genotype to cheese yield and profit (Tables 8 and 9) were based on current market prices as of March 2004 (Table 1). Unfortunately, the price of cream was very high relative to other inputs and products, thus making it difficult to directly assess the value of milk composition and genotype on dried whey products. The only products that were profitable to produce were cheese, fat-reduced whey sold, and cream sold. Virtually no dried whey products were produced. If the price of cream had been lower, more products would have been produced (Craig et al., 1989). Therefore, many simulations were run, with the costs of NDM and cream bought adjusted to find the maximum price the plant would be willing to pay for more protein and fat, respectively, and the minimum selling price for demineralized whey, 34% whey protein concentrate, and 80% whey protein concentrate. Comparing market prices used (Table 1)

to the value of purchased inputs to the plant estimated by the model (Tables 10 and 11) revealed that NDM was overpriced at \$1.77/kg market and needed to be reduced to \$1.50/kg, based on the estimated model value (average of Table 10 excluding the CN composition row and the AA2F genotype column). Cream bought was also overpriced at \$4.16/kg market and needed to be reduced to \$1.38/kg, based on estimated model value (average of Table 11 excluding the low-fat composition row and the high-CN composition-AA2F genotype value). Similarly, demineralized whey was underpriced at \$1.25/kg market (Table 1) and needed to be increased to \$12.50/kg, based on the estimated model value (average of Table 12); 34% whey protein concentrate was underpriced at \$1.61/kg market and need to be increased to \$26.66/kg, based on the estimated model value (average of Table 13); and 80% whey protein concentrate was underpriced at \$4.74/kg mar-

Table 13. Minimum prices (\$/kg) willing to sell 34% whey protein concentrate produced from a cheese-processing plant for each milk composition¹ and genotype combination²

Milk composition	Genotype										Average ³
	AA1P	BB1P	AVG1	AA2P	BB2P	AVG2	AA1F	BB1F	AA2F	BB2F	
Average 3.25% protein	23.90	23.09	23.89	23.78	23.90	23.90	23.90	23.90	23.90	23.90	23.81
Low WP 3.25% protein	21.03	21.21	21.11	23.06	23.26	23.26	21.17	21.06	23.26	21.90	22.03
High WP 3.25% protein	42.43	42.43	42.44	42.26	42.21	42.23	42.39	42.45	42.03	42.40	42.33
Low CN	19.20	19.32	19.06	20.46	20.77	20.65	19.29	19.21	21.57	19.74	19.93
High CN	23.39	23.39	23.39	23.39	23.39	23.39	23.39	23.39	23.39	23.39	23.39
Low WP	42.83	42.76	42.81	42.80	42.81	42.81	42.76	42.83	42.74	42.76	42.79
High WP	23.14	23.14	23.14	23.09	23.14	23.15	23.14	23.14	23.14	23.09	23.13
Low fat	24.53	24.52	24.52	24.52	24.53	24.52	24.52	24.52	24.53	24.53	24.52
High fat	22.78	22.76	22.79	22.74	22.79	22.78	22.78	22.76	22.79	22.79	22.78
Low lactose	25.18	25.18	25.18	25.16	25.17	25.16	25.17	25.17	25.18	25.18	25.17
High lactose	23.39	23.40	23.39	23.40	23.35	23.39	23.39	23.39	23.40	23.39	23.39

¹See Table 4.²Symbols represent different genotype combinations of CN and fat retentions from Table 7 used in simulations. WP = Whey protein.³Average across columns.

Table 14. Minimum prices (\$/kg) willing to sell 80% whey protein concentrate produced from a cheese-processing plant for each milk composition¹ and genotype combination²

Milk composition	Genotype										Average ³
	AA1P	BB1P	AVG1	AA2P	BB2P	AVG2	AA1F	BB1F	AA2F	BB2F	
Average 3.25% protein	54.08	54.09	54.05	54.05	53.78	54.08	54.08	54.08	54.03	54.08	54.04
Low WP 3.25% protein	33.12	33.30	33.23	34.98	35.16	35.16	33.29	33.15	35.16	33.95	34.05
High WP 3.25% protein	97.83	97.84	97.85	97.43	97.32	97.38	97.85	97.89	96.89	97.78	97.61
Low CN	43.00	43.31	42.70	46.09	46.72	46.42	43.23	43.05	48.65	44.30	44.75
High CN	52.89	52.89	52.90	52.91	52.92	52.91	52.90	52.89	52.93	52.91	52.91
Low WP	98.77	98.61	98.77	98.72	98.73	98.71	98.60	98.77	98.58	98.50	98.68
High WP	34.90	34.90	34.90	34.82	34.91	34.91	34.89	34.91	34.94	34.90	34.90
Low fat	55.53	55.55	55.54	55.54	55.54	55.54	55.55	55.54	55.56	55.53	55.54
High fat	51.36	51.30	51.46	51.35	51.45	51.44	51.46	51.30	51.47	51.46	51.41
Low lactose	57.10	57.10	57.10	57.05	57.07	57.07	57.08	57.07	57.09	57.08	57.08
High lactose	52.92	52.90	52.88	52.90	52.81	52.87	52.88	52.88	52.90	52.88	52.88

¹See Table 4.²Symbols represent different genotype combinations of CN and fat retentions from Table 7 used in simulations. WP = Whey protein.³Average across columns.

ket and needed to be increased to \$57.62/kg, based on the estimated model value (average of Table 14). Different market prices would result in different values for the 2 inputs and 3 products.

Other models have been developed for improving cheese plant production and efficiency. The Van Slyke equation is the best known and most widely used (Covington, 1998). The Van Slyke equation predicts cheese yield based on the percentages of protein, fat, and moisture. The Van Slyke equation has been incorporated into other mass balance linear programming models to specifically address the standardization of milk inputs to obtain a consistent product. A linear mass balance optimization model by Kerrigan and Norback (1986) examined the results of using different types of objectives functions: to maximize net returns, to maximize cheese yield, or to minimize cost. Craig et al. (1989) built on the model developed by Kerrigan and Norback (1986) to maximize net returns from natural cheese and the use of intermediate products (cheese, whey cream, and separated whey) from cheese making to make processed cheese products. But in both models, they did not consider components other than protein, fat, and moisture, the impact of different milk compositions on cheese making, and the use of retentions to formulate intermediates and products to evaluate genotype or component differences or the production of dried whey products. Papadatos et al. (2002) developed an optimization mass balance model maximizing net revenue in cheese making and examined changes in milk price and milk composition on Cheddar cheese and low-moisture part-skim Mozzarella. The model was similar to models by Kerrigan and Norback (1986) and Craig et al. (1989) because they used an equation such as the Van Slyke equation or Barbano (Papadatos et al., 2002) to estimate cheese yield instead of retentions. Unlike previous

models, however, products such as whey protein concentrates, whey powder, and lactose powder were included in the model reported here. The current model provides a structure that allows retention data on individual input compositions to be evaluated in an optimization format.

CONCLUSIONS

The model developed is unique in its ability to track the components of milk and other purchased inputs and to estimate yields through the retentions of components. The results demonstrated that milk composition has a greater effect on Cheddar cheese production and profit than does genotype. The model estimated that a high-CN milk composition (3.57%) would result in the greatest cheese yield (9,846 kg/100,000 kg of milk) but that a high-fat milk composition (5.97%) would result in the greatest profit (\$70,586/100,000 kg of milk). The AA and BB κ -CN genotypes were assumed to primarily affect the retention of milk components in cheese. Although the κ -CN B allele has been associated with increased Cheddar cheese yield, the model estimated that the increased cheese yield was not significantly different between the AA and BB κ -CN genotypes and would not significantly increase profit. Therefore, a cheese plant would not benefit from encouraging dairy producers to select for bulls with the B κ -CN allele at the prices and retentions examined using the model. More data on how milk expressing the B variant of κ -CN affects retentions, cheese yield, or both may change these results.

The model provides managers of a cheese plant with the ability to optimize component use, maximize profit, and assess cheese production data and efficiency. With a trend toward increased whey by-product processing

and utilization, the model will be of benefit for setting prices on several whey products in integrated plants.

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