Short communication: Estimating lactation curves for highly inhomogeneous milk yield data of an F₂ population (Charolais × German Holstein)

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ABSTRACT

Fitting of lactation curves is a common tool to obtain the entire milk yield as well as to estimate the main curve characteristic (such as day of peak milk yield) for a lactation. These models are primarily designed for dairy cattle, but have been applied to nondairy cattle breeds and also for other species. In this study we considered milk yield data of 197 F₂ crossbred cows of Charolais and German Holstein (founder breeds) for the first and the beginning of the second lactation. The F₂ cows showed a high variability regarding the length of lactation, which varied between 7 and 406 d in milk for the first lactation. Thus, the data also show high variation regarding the daily and overall milk yield. To obtain complete lactation curves, we evaluated the lactation models of Ali-Schaeffer and Wilmink. To compare the 2 lactation models, we evaluated the goodness of fit using 6 evaluation criteria. The results show that the model of Ali-Schaeffer performs better on these highly inhomogeneous data, in contrast to the model of Wilmink. We discuss our findings from a statistical point of view and present possible biological reasons for the high variability regarding milk yield within the F₂ population. Hence our findings may be helpful when milk yield data of crosses between dairy and beef cows (dual purpose) are investigated, whose lactation curves may not show the typical characteristics of dairy cattle. **Key words:** crossbred population, milk yield, lactation model, goodness of fit

Short Communication

Lactation curves are mainly used (1) to evaluate the time course of milk production, (2) to investigate and understand the genetic potential for milk production for a breed, and (3) as a helpful tool to improve management (e.g., Macciotta et al., 2011; Vinay-Vadillo et al., 2012). For fitting lactation curves, several lactation

models are proposed in literature (for a review see Macciotta et al., 2011). The most common lactation models are the model of Wood (Wood, 1967), Ali-Schaeffer (AS; Ali and Schaeffer, 1987), and Wilmink (WM; Wilmink, 1987), which are often applied in such studies (e.g., Jenkins and Ferrell, 1992; Macciotta et al., 2011). In general, the goodness of fit for each model depends on the underlying structure of the data, such as duration of lactation, shape of the lactation curve, or homogeneity of investigated data (Pulina et al., 2005). Lactation models have not only been applied to data that show the typical characteristics of lactation curves, but also to data that include atypical lactation curves (e.g., curves without a lactation peak; Congleton and Everett, 1980; Macciotta et al., 2011). Examples for such curves exist in various crossbred populations, for example, Holstein × Zebu (Vinay-Vadillo et al., 2012) or Ayrshire × (Sahiwal × Ankole) (Hatungumukama et al., 2007), other species, for example, sheep (Bilgin et al., 2010), or the curves obtained for other milk ingredients, like the ratio of fat:protein content (Buttchereit et al., 2010). In literature, several lactation models are compared using data that include atypical curves, whereby it was often observed that the model of AS was the best (e.g., Buttchereit et al., 2010; Vinay-Vadillo et al., 2012). Moreover, it is known that the model of Wood is sensitive regarding data distribution (e.g., Macciotta et al., 2005; Silvestre et al., 2006). Hence, we investigated the applicability of the 2 established lactation models of (WM, AS) on an experimental F₂ population that shows high variety in milk production.

The founders of the experimental F₂ population were 5 Charolais bulls and 47 German Holstein cows. The offspring of these founder animals was crossed to establish the F₂ population (SEGFAM project; Kühn et al., 2002). Both, the F₁ and F₂ generations were raised under standardized dairy conditions at the farm of the Leibniz Institute for Farm Animal Biology (FBN, Dummerstorf, Germany). Milking data for 197 F₂ cows were available for the first and the beginning of the second lactation (recorded between 2003 and 2013). All cows changed from a tie-stall barn to a free-stall barn at the beginning of both lactations. The cows were always milked twice daily (at approximately 0300–0400 and

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1500–1600 h) in a conventional milking parlor, where the milk yield was measured via commercial milk meters (ALPRO, DeLaval, Tumba, Sweden). Milking data for each cow were recorded until the cows dried off themselves or, in case of the first lactation, gave less than 2 kg of milk after 110 DIM. After the second calving, most of the cows were slaughtered around 30 DIM, and the remaining cows were slaughtered around 100 DIM for experimental reasons. Calves were separated from their dams shortly after calving and thus were not allowed to suckle their dams in both lactations. The F_2 cows with long lactations were inseminated starting at 201 DIM, whereas cows with shorter lactations were inseminated after they were dried off. The experimental procedures were carried out with respect to the animal care guidelines of the State Mecklenburg-Western Pomerania, Germany, and were approved by the Committee for Animal Use and Care of the Ministry of Agriculture, the Environment and Consumer Protection of the Federal State Mecklenburg-Western Pomerania, Germany.

We used all daily milk yields (i.e., the sum of the morning and evening milking) starting at 1 d after calving (according to ADR, 2008) until our defined end of each lactation. A defined end is necessary because the experimental F_2 cows were milked as long as possible. As result, various cows had at the end of lactation very low daily milk yields (<0.5 kg/d) over several days, especially before 110 DIM in the first lactation. Hence we introduced the following criteria to enable a higher degree of comparability. A lactation was defined as finished (1) if the daily milk yield was under 0.5 kg/d for a second consecutive time, whereby it was ensured that these 2 d were not just a drop down within the lactation by checking the values of the consecutive 2 d and the corresponding 2 d before or (2) the 305th day of lactation was reached in the first lactation or 1 d before slaughter in the second lactation. The threshold of 0.5 kg/d was not applied before the 5th day of lactation to ensure that the increasing slope of the lactation curve was not affected. For the first lactation we excluded 8 F_2 cows because of too many consecutively missing daily milk yields. Three of the 197 F_2 cows had no second lactation. Individual daily milk yields were excluded (set to “NA”) for various reasons such as medical treatment of a cow, suckling by another cow, or due to technical problems. If one milk measurement was missing for the day then the corresponding daily milk yield was set to “NA.” This preprocessing unifies the end of lactation for the F_2 cows and thus improves the fit of lactation curves. However, a general recommendation how to set this cut-off value when milk yield data of crossbreeds are considered cannot be given. After preprocessing we had durations of lactation in the interval of 7 to 305

d (average 160.8 ± 117.0 DIM; missing values $3.2 \pm 4.8\%$) for the first lactation. For the second lactation we had durations of lactation in the interval of 15 to 103 d (slaughter; on average 33.6 ± 17.6 DIM; missing values $1.8 \pm 3.3\%$).

In this study both lactations were considered separately. The following random regression model was applied to the observed daily milk yields for each lactation:

$$y_{ijklm} = \beta \cdot age_i + year_j \times season_k + \sum_{n=1}^4 b_n \cdot s_{ijklmn} \langle d \rangle + \sum_{n=1}^4 cow_{ln} \cdot s_{ijklmn} \langle d \rangle + \varepsilon_{ijklm}, \quad [1]$$

where y_{ijklm} is the m th daily milk yield of the l th cow taken at lactation day d . We consider the following fixed effects related to calving day: the linear regression on age_i in d [$i \in (739, \dots, 1,126)$ first lactation; $i \in (1,096, \dots, 1,752)$ second lactation], where β is the corresponding regression coefficient and the interaction between $year_j$ ($j = 2003, \dots, 2011$ first lactation; $j = 2004, \dots, 2013$ second lactation) and $season_k$ ($k = \text{winter, spring, summer, autumn}$), which resulted in 27 levels for the first lactation and 32 levels for the second lactation. s_{ijklmn} represents the corresponding model terms for the applied lactation model:

$$\text{AS: } s_{ijklm0} = 1, \quad s_{ijklm1} = \frac{d}{305}, \quad s_{ijklm2} = \left(\frac{d}{305}\right)^2, \quad s_{ijklm3} = \ln\left(\frac{305}{d}\right), \quad s_{ijklm4} = \ln\left(\frac{305}{d}\right)^2,$$

$$\text{WM: } s_{ijklm0} = 1, \quad s_{ijklm1} = d, \quad s_{ijklm2} = e^{-0.05 \cdot d},$$

where b_n represents the fixed regression coefficient for each model term for the corresponding lactation model on lactation day ($d = 1, \dots, 305$ first lactation; $d = 1, \dots, 30$ second lactation). The intercept is already included within $year \times season$ (and regression on age), and hence the constant of s_{ijklm0} is not necessary. To model cow-specific lactation curves and to account for covariances between daily milk yields for the same cow, we added further random regression coefficients. In Equation [1], cow_{ln} represents the random regression coefficient for each model term for the corresponding lactation model on lactation day d of the l th cow. ε_{ijklm} represents the random residual effect. In addition, changing the barn was not considered as a fixed effect because all F_2 cows had a change at the beginning of lactation in both lactations.

The random regression model was analyzed using the SAS procedure GLIMMIX (version 9.3, SAS Institute Inc., Cary, NC) and an unstructured covariance matrix was used to model the dependencies between test days. We decided to use lactation d 1 until 305 for modeling of the individual lactation curves for the first lactation, because 29% of F₂ cows have more than 300 DIM. In the second lactation, we used lactation d 1 until 30 for modeling of the individual lactation curves, because we observed a steep decrease in number of observations from 50 to 7% between d 30 and 40 as most cows were slaughtered at that time point. To determine the goodness of fit with respect to model complexity, we used the Akaike information criterion (**AIC**; Akaike, 1974) and Bayesian information criterion (**BIC**; Schwarz, 1978). For those 2 criteria, the lactation model that shows the lowest values is best. While AIC and BIC are determined over all fitted lactation curves, the following 4 criteria are determined for each fitted curve individually. As third criteria we determined the Pearson correlation coefficient between observed and predicted daily milk yield (**r**) to quantify the degree of association between observed and predicted values as proposed by Guo and Swalve (1995). Fourth, we calculated the mean squared prediction error (**MSPE**) to obtain how well the regression model fits the observed daily milk yields. The MSPE is determined as follows (Hocking, 1976):

$$\text{MSPE} = \frac{\sum_{t=1}^q (y_t - \hat{y}_t)^2}{q - p}, \quad [2]$$

where y_t is the observed milk yield on lactation day t and $\hat{y}_t$ is the corresponding predicted milk yield. Further, q is the total number of observed daily milk yields for each cow and p is the number of parameters of the lactation model used ($p = 3$ for WM and $p = 5$ for AS). The model that results in the lowest MSPE value is best. The similarity between measured and estimated lactation curve (**Q**) is determined as follows (Ali and Schaeffer, 1987):

$$Q = 100 \cdot \frac{\sum_{t=1}^q e_t^2}{\sum_{t=1}^q y_t^2}, \quad [3]$$

where e_t is the residual for lactation day t . All other variables are defined as in Equation [2]. Here the lowest values represent the best model. Finally, as test for serial first-order autocorrelation, the Durbin-Watson

coefficient (**DW**; Durbin and Watson, 1951; Dufour and Dagenais, 1985) is determined:

$$\text{DW} = \frac{\sum_{t=2}^q (e_t - e_{(t-1)})^2}{\sum_{t=1}^q e_t^2}. \quad [4]$$

A DW coefficient of 2 means no autocorrelation, whereas a DW coefficient close to 0 or 4 means a positive or a negative autocorrelation, respectively. As general rule, DW coefficients between 1.5 and 2.5 are considered to have no autocorrelation (Koncagul and Yazgan, 2008; Ott and Longnecker, 2010). Further, the residuals for each DIM were averaged and plotted to enable a visual inspection for autocorrelation. The evaluation criteria were determined in R (R Core Team, 2013), except AIC and BIC, which were obtained by SAS.

Figure 1 shows fitted lactation curves for the first lactation resulting from the models of AS and WM for 10 example F₂ cows. These selected lactation curves mainly reflect the shapes of lactation curves within the data set and even show the variability regarding milk yield within the F₂ cows. The estimated shape of lactation curves based on the model of AS reflect the observed curves based on measured milk yields generally better than WM, especially at the onset of lactation (Figure 1, examples 2 and 9); AS models the increasing slope well, and WM does not. In addition, in Supplemental Figure S1 (<https://doi.org/10.3168/jds.2017-12772>), we plotted all fitted lactation curves for the first lactation. The better overall fit of AS is further confirmed by the results obtained using the evaluation criteria. Table 1 lists the values for the 6 applied evaluation criteria for AS and WM for both lactations. The values show that the model of AS performs better compared with WM in both lactations. Further, the DW coefficients computed for the first lactation indicate that both models exhibit positive autocorrelation within the residuals, although the autocorrelation is lower for the AS model than for the WM model. For the second lactation, hardly any autocorrelation for the AS model is detected, whereas the model of WM has positive autocorrelations within the residuals. This observation is further supported by the visual inspection of the averaged residuals for each lactation day for both lactations over the corresponding investigated time frame. In Figure 2A the averaged residuals show a similar trend for AS and WM for the first lactation. In contrast, for the second lactation (Figure 2B), no obvious trend is observed for the model of AS, whereas a clear trend is present for the model of WM. Further, it can be seen that especially the milk

Figure 1. For the first lactation, fitted lactation curves resulting from the lactation models of Ali-Schaeffer (AS) and Wilmlink (WM) in comparison to the observed daily milk yields (Obs) for 10 F_2 cows. Color version available online.

yield at the first day of lactation is strongly underestimated by WM. In detail, WM underestimates the daily milk yield on average by 2.6 kg in the first lactation and by 3.4 kg in the second lactation. In contrast, AS underestimates the daily milk yield on average only by 0.2 kg in the first lactation and has an almost exact estimation in the second lactation. In general, our results show that AS fits to the highly diverse data of the F_2 cows better than WM regarding the applied evaluation criteria, which is further supported by visual inspection of the fitted curves (Figure 1, Supplemental Figure S1, <https://doi.org/10.3168/jds.2017-12772>). The reason for this result may be that the higher number of parameters for AS (5) allows a better fit of the lactation curves in comparison to WM (3). As Macciotta et al. (2005) pointed out, WM may model 4 different shapes of lactation curves, whereas AS is able to model 32 different shapes. This is advantageous in our case, as the examples in Figure 1 show. Buttchereit et al. (2010) also came to the conclusion that lactation models with a fewer number of parameters may not be able to model all variants of the curves.

Apart from the presented random regression model, we also tested alternative models (1) without fixed effect $year \times season$, (2) only $year$ effect, (3) only $season$ effect, and (4) interaction of $year \times season$ where $season_k$ is divided in (k = winter, summer). Based on the evaluation criteria, the models with 4 $season_k$ performed best, and the alternative model (4) is second best. We also evaluated fitting lactation curves for each F_2 cow individually using fixed regression. Fitting lactation curves for cows individually was not appropriate for our inhomogeneous data, as especially in cases of missing daily milk yields at the beginning or end of the lactation, both models often strongly under- or overestimated daily milk yields. Because our population of F_2 cows exhibits a wide variety of lactation curves, determining typical characteristics for an entire group of cows, such as peak day of lactation or persistency of milk production, is not meaningful. These characteristics may only be determined for individuals. For the first lactation, the range of total milk yield for F_2

cows that reached 305 DIM was estimated to be between 1,591 and 5,359 kg based on the model of AS. To classify these values, it is reported in literature that German Holstein cows may produce well over 9,000 kg of 305 DIM (Duske et al., 2009). In contrast, for Charolais it is reported to be about 1,500 kg for 205 DIM (Jenkins and Ferrell, 1992). Generally the F_2 cows show high variability in milk production in both lactations, indicating that some F_2 cows inherited more beef type characteristics, whereas others inherited more dairy type characteristics. This is further supported by the observed external phenotype such as BW and carcass composition (Hamada et al., 2012). Further, Hammon et al. (2010) found a relation between milk production and udder size in 2 families of these F_2 cows, whereby udder size is only a partial explanation for increased milk yield in dairy cows (Keys et al., 1989). In general, dairy breeds, such as Holstein, have been selected for milk production. Suckling, which is a natural stimulation for milk letdown, has been completely replaced by hand or machine milking. In contrast, beef breeds, such as Charolais, have mainly been selected for high daily BW gain, calf weaning weight, and high degree of muscularity (e.g., Cunningham, 1974; O'Neill et al., 2010). Beef cows are mainly suckled by their calves and rarely milked. In our case the calves were separated from the F_2 cows shortly after calving. This fact may also be responsible for poor milk yield in parts of the F_2 population, as it has been reported that the presence of the calf plays an important role for milk letdown [e.g., for cow-calf pairs of Angus $\times$ Hereford (Lamb et al., 1999) or Ayrshire $\times$ (Sahiwal $\times$ Ankole) cattle (Hatungumukama et al., 2007)]. This is further supported by the study of Alvarez et al. (1980), who used crossbreeds between European (Holstein, Brown Swiss; *Bos taurus*) and Zebu cows (*Bos indicus*). Cows with restricted suckling had significantly longer lactation (287 ± 11 d) than cows without calf contact (73 ± 26 d), although they had similar DIM in a previous lactation. So far, most studies with highly varying milk yield data only involve crossbreeds between *Bos taurus* and *Bos indicus*. In contrast, we had milk data of a

Table 1. Results for the applied evaluation criteria are listed for both lactations (corresponding SD in parentheses)¹

Model	AIC	BIC	r	MSPE	Q	DW
Lactation 1						
Ali-Schaeffer	91,909	91,961	0.842 (0.116)	0.967 (0.422)	4.897 (4.937)	1.373 (0.431)
Wilmink	98,503	98,526	0.792 (0.142)	1.081 (0.455)	6.372 (5.888)	1.136 (0.427)
Lactation 2						
Ali-Schaeffer	20,842	20,849	0.871 (0.112)	1.248 (0.487)	1.481 (3.033)	2.122 (0.495)
Wilmink	23,241	23,264	0.734 (0.174)	1.705 (0.576)	2.801 (4.263)	1.347 (0.511)

¹The numbers in bold represent the best values for the given criteria. AIC = Akaike information criterion; BIC = Bayesian information criterion; r = correlation between observed and predicted milk yield; MSPE = mean squared prediction error; Q = similarity between observed and predicted lactation curve; DW = Durbin-Watson coefficient.

Figure 2. Averaged residuals for each DIM for the first (A) and second (B) lactation resulting from the lactation models of Ali-Schaeffer (AS) and Wilink (WM). Color version available online.

cross of 2 *Bos taurus* breeds, which shows similarities to previous work. Hence it is possible that for some of the F_2 cows (especially for cows that inherited more of the beef type characteristics), the absence of their calves may have a negative effect on their milk production.

In conclusion, even for such highly inhomogeneous lactation data as may be observed in various crossbreed populations (dual purpose), established lactation models may be used. For our crossbreed population of Charolais and Holstein, the model of Ali-Schaeffer worked best.

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