Optimal Replacement of Mastitic Cows Determined by a Hierarchic Markov Process

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ABSTRACT

Farmers frequently have to decide whether to keep or to replace cows that suffer from clinical mastitis. A dynamic programming model was developed to optimize these decisions for individual cows within the herd, using the hierarchic Markov process technique. This technique provides a method to model a wide variety of cows, differing in age, productive performance, reproductive status, and clinical mastitis occurrence. The model presented was able to support decisions related to 63% of all replacements. Results—for Dutch conditions—showed the considerable impact of mastitis on expected income of affected cows. Nevertheless, in most cases, the optimal decision was to keep and to treat rather than to replace the cow. Clinical mastitis occurring in the previous lactation negligibly influence expected income. Clinical mastitis in current lactation, especially in the current month, however, had a significant effect on expected income.

Total losses caused by clinical mastitis were US$83/yr per cow. Farm level treatment, which reduced incidence by 25%, on a farm with 10 clinical quarter cases per 10,000 cow days, may cost at maximum US$27/yr per cow.

(Key words: economics, dynamic programming, mastitis, replacement)

Abbreviation key: CQ = accumulated number of diagnosed clinical quarters in the current lactation from the beginning of the lactation (t = 0) through month t - 1 (four levels: 0, 1, 2, and ≥3). DP = dynamic programming. DQ = binary variable indicating that clinical mastitis was diagnosed in at least one quarter (≥1) or in none of the quarters (0) in month t of the current lactation. HMP = hierarchic Markov process, PQ = accumulated number of CQ in the previous lactation (four levels: 0, 1, 2, and ≥3).

INTRODUCTION

A farmer’s replacement policy of dairy cows greatly influences profitability (2, 19). According to Morris and Marsh (18) and Van Arendonk (25), major reasons for culling cows are low production, failure to conceive, and mastitis. Mastitis has a large economic impact and is considered to be the most important health problem in many countries (20). Houben et al. (8) analyzed 5313 lactation of 2477 Dutch Black and White cows and described estimated production parameters and probabilities of clinical mastitis reoccurrence. Over a mid to long-term period (>1 mo), clinical mastitis reduced milk production by 2.3 to 6.2%. Clinical mastitis appears to be repetitive across lactations and, therefore, has important economic implications, which in turn affect decision making. Decisions to replace cows are mainly based on economic rather than biological considerations; i.e., the farmer expects to improve profits by replacing the cow (25). The inherent biological cycles of reproduction and lactation make dairy cow management decisions dynamic, recursive, and stochastic. Replacement decisions are in the first place time-dependent. In recursive stochastic multistage optimization problems, dynamic programming (DP) has the advantage of deter-
mining optimal decisions without requiring exhaustive enumeration of all sequences of transition possibilities (3).

Jenkins and Halter (12), Giaever (6), and Smith (21) used the DP approach in their comprehensive studies to optimize dairy cow replacement policies. Before the 1980s, detailed DP models were not available to support decisions because of the lack of computer capacity. Subsequently, several DP models were developed to optimize decisions to replace individual animals (cows and sows) based on production capacity, reproductive status, or both (3, 10, 13, 15, 22, 24, 26).

Stott and Kennedy (22) included clinical mastitis as a state variable in their replacement model. Replacement within a lactation was not possible in their model, and mastitis state was treated as a binomial variable rather than a multilevel variable. The model for clinical mastitis was not detailed enough to permit economic conclusions.

In the Netherlands, the main reasons for culling cows are poor production and appearance (35%), poor fertility (20%), and mastitis (8%) (27). If these factors were successfully included in a replacement model, the system would support about 63% of all replacement decisions in dairy cattle management. Because risk factors for clinical mastitis include production, age, month in lactation, and mastitis history (8), and because of the stochastic nature of mastitis, addition of clinical mastitis to replacement and insemination models appears to be worthwhile.

One objective of this paper was to develop a DP model for dairy cows insemination and replacement decisions that include, in a detailed way, clinical mastitis as a state variable. Another objective was to carry out a sensitivity analysis to gain insight into the relationship between parameters and losses related to clinical mastitis. Methods used provide a means of determining actual costs of clinical mastitis and maximum cost of farm level treatments on an average farm.

**MATERIALS AND METHODS**

**Markov Decision Theory**

The general description of Markov decision theory and the basic formulation of the hierarchical Markov process as presented in this and the next section were mainly based on the publication by Kristensen (14).

Consider a time-dependent Markov decision process with a finite state space $U$ at each stage $t$ and a finite decision set $D$. Policy $s$ is a map assigning to each state $i$ at stage $t$ a decision $d(i,t) \in D$. The time interval between two transitions is called a stage ($t$). Let $p_{ij}(t,d)$ be the transition probability from state $i$ to state $j$ if decision $d$ is taken at stage $t$. If in state $i$ at stage $t$ a decision $d$ is chosen, then (according to the Markovian property), regardless of the history of the system, 1) an immediate expected reward $r_i(t,d)$ is obtained, and 2), at the next stage, the system will be in state $j$ with probability $p_{ij}(t,d)$. According to the Markovian property, the immediate expected reward obtained from the decision made in state $i$ at stage $t$ is not dependent upon the next state $j$. This general Markov decision process is reviewed at equidistant points in time $t$. However, semi-Markov decision theory assumes that, if state $i$ is observed and decision $d$ is made, a physical quantity (e.g., time or milk) of $m_i(t,d)$ is involved in the transition of the system.

An optimal policy is defined as a policy that maximizes (or minimizes) some predefined objective function. The objective function can maximize the total expected discounted rewards over the planning horizon (discounting criterion) or the expected average reward per unit of time (average criterion). To solve general Markov decision problems by DP, value and policy iteration can be used as optimization techniques. With value iteration, the optimal policy is determined sequentially using the recurrent equations of Bellman (1). Value iteration is exact when optimization occurs under a finite planning horizon. Under an infinite planning horizon, however, the value iteration method can be used to approximate the optimal policy, especially in case of cyclic production. Value iteration makes it possible to handle large models. With policy iteration, a set of linear simultaneous equations are solved (9). By discounting criteria, policy iteration determines the total present value of the expected future rewards of a process starting in a certain state under a given policy. When an average criterion is used, the set of simultaneous equations determines the relative value of...
OPTIMAL REPLACEMENT OF MASTITIC COWS

2977

each state and, moreover, the average reward per unit of time (gain) under a certain policy. With this information, a new policy is chosen, which maximizes the objective function. Those steps are performed iteratively until the policy no longer changes. Policy iteration can only be used for optimization under an infinite planning horizon and is exact in that case. Because of the more complicated mathematical formulation involving a solution of large systems of simultaneous linear equations, the method can only handle rather small models (17).

Kristensen (14) developed an alternative structure of a Markov process, called the hierarchic Markov process (HMP), that includes both value iteration and policy iteration in one model. In our study, the HMP approach was used.

**HMP**

One of the reasons that replacement models, formulated as a general Markov decision process, are usually very large is that the age of the animal in question is included as a separate state variable (14). As a result, most elements of the transition matrix equal zero because these transitions are not feasible (e.g., immediate transition from the second to the fifth lactation is not possible). The HMP omits age as a state variable and, moreover, takes advantage of the fact that when a replacement occurs the process (life cycle of the replacement animal) is restarted. In the traditional Markov decision model, a replacement is represented just as a transition from one state to another. In an HMP, the general Markov decision process is split into one main process and subprocesses. Each state in the main process represents a separate Markov decision process (a subprocess) with a finite number of stages (i.e., the maximum lifespan of a cow). The structure of the transition matrix of an HMP is shown in Figure 1.

The number of subprocesses equals the number of states in the main process. State variables of the main process concern states of the cow that do not change during its life time (e.g., age at first calving). The immediate expected rewards (net revenue from a single stage) in the main process are calculated from the rewards of the subprocesses. The time step (stage duration) in the main process equals the total length of the corresponding subprocess (13).

One advantage of an HMP over a general Markov decision process is that the number of transition probabilities is reduced by a factor equal to the square of the number of aging states. If, for instance, 12 lactations are distinguished in the model, the number of transition probabilities is reduced by $12^2$. However, this reduction is not really a reduction of the number of transition probabilities because these probabilities are all zero in a general Markov decision process. In other words, the HMP especially refers to the nonzero part of the transition matrix of a general Markov decision process.

The main advantage of the HMP is directly related to its structure. A subprocess has a well-defined and finite planning horizon (lifespan of cow). This and its large state space make value iteration the ideal optimization method to use. The main process has a small state space and an infinite planning horizon; therefore, policy iteration can be applied without computation problems. It can be proved mathematically that the complete HMP should be regarded as a general Markov decision process optimized with policy iteration (14). Application of the value iteration method in the subprocesses with a finite planning horizon and policy iteration in the main process with an infinite planning horizon results in a sound optimization technique that is fast, exact, and able to handle very large models (14, 16).

In our study, a special case of the HMP was used with only one state in the main process (i.e., one type of subprocess). The objective function in the current cow replacement model was to maximize average net revenues per time unit. Discounting of net revenues was not applied because previous work (16) showed that discounting had no effect on the optimal strategy.

**Optimization of the HMP**

The iterative optimization procedure of the HMP starts with the choice of an arbitrary policy. In the second step, this policy is used to calculate the total expected reward and total expected output from the remaining part of the process for each state at each stage in a subprocess. Weighing of the total expected reward

and output at stage 1 with the start distribution of a subprocess produces the total expected reward and total expected output for each state in the main process. With this information, the relative value of each state in the main process and the gain can be calculated by using matrix algebra. In the last step of the procedure, the gain is used to determine the new improved

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Figure 1. Transition probability structure of a hierarchic Markov process (T is the maximum age of a cow).

policy. Steps 2 and 3 are repeated until the policy is stable, i.e., does not change with further iterations. These steps are described in formula form within the following paragraphs and are based on work by Kristensen (16).

In the notation, \( \alpha \) and \( \beta \) are used to denote states of the main process and \( i \) and \( j \) to denote states of subprocesses. A policy for a subprocess is denoted \( s \), and the map of policies of subprocesses is denoted \( \sigma \) (i.e., \( s = \sigma(\alpha) \)). The three steps of the iteration cycle are

**Step 1.** Choose an arbitrary policy \( \sigma \). Go to step 2.

**Step 2.** Solve the following set of \( u + 1 \) linear simultaneous equations for \( g(\sigma) \) and \( F_\alpha(\sigma) \) by using matrix algebra:

\[
g(\sigma)h_\alpha(\sigma) + F_\alpha(\sigma) = \\
f_\alpha(\sigma) + \sum_{\beta = 1}^{u} P_{\alpha\beta}F_\beta(\sigma),
\]

where

- \( g(\sigma) \) = gain (i.e., reward per physical output) under policy \( \sigma \);
- \( F_\alpha(\sigma) \) = relative value of state \( \alpha \) under policy \( \sigma \);
- \( P_{\alpha\beta} \) = transition probability from state \( \alpha \) to state \( \beta \) in the main process;
- \( u \) = number of subprocesses;
- \( f_\alpha(\sigma) \) = the reward in state \( \alpha \) of the main process

\[
= \sum_{i=1}^{I_t} p_i(0)f_i(1,s)
\]

(the right-hand side of this equation belongs to subprocess \( \alpha \));

- \( h_\alpha(\sigma) \) = the physical output in state \( \alpha \) of the main process

\[
= \sum_{i=1}^{I_t} p_i(0)h_i(1,s)
\]

(the right-hand side of this equation belongs to subprocess \( \alpha \));

- \( p_i(0) \) = probability of starting at state \( i \) in a subprocess \( \alpha \);

\[
f_i(t,s) = \text{total expected reward from the remaining part of the process under policy } s \text{ if present state and stage are } i \text{ and } t, \text{ respectively},
\]

\[
r_i(t,s) = \text{immediate expected reward in state } i \text{ at stage } t \text{ under policy } s;
\]

\[
p_{ij}(t,s) = \text{transition probability from state } i \text{ to state } j \text{ where } i \text{ is the state of the stage } t, j \text{ is the state of the following stage, and } s \text{ is the policy};
\]

\[
I_t = \text{number of states at stage } t;
\]

\[
h_i(t,s) = \text{total expected output from the remaining part of the process under policy } s \text{ when present state and stage are } i \text{ and } t, \text{ respectively},
\]

\[
m_i(t,s) = \text{immediate physical output in state } i \text{ at stage } t \text{ under policy } s,
\]

Go to step 3.

**Step 3.** For each subprocess \( \alpha \), find by means of the recurrence equations a policy \( s' \) of the subprocess:

\[
V_{\alpha}(t) = \max_{d \in D} \left\{ r_i(t,d) - m_i(t,d)g(\sigma), \quad t = T \right\}
\]

\[
= \max_{d \in D} \left\{ r_i(t,d) - m_i(t,d)g(\sigma) + \sum_{j=1}^{I_{t+1}} p_{ij}(t,d)V_{\alpha j}(t+1) \right\},
\]

\[
V_{\alpha}(t) \quad \text{where } V_{\alpha j}(t) = \text{maximum relative value at subprocess } \alpha, \text{ state } j, \text{ and stage } t.
\]

The decision \( d'(t,i) \) maximizes the right-hand side of the recurrence equation of state \( i \) at stage \( t \). Those decisions determine the new policy \( s' \). Put \( \sigma'(\alpha) = s' \) for \( \alpha = 1, \ldots, u \). If the new policy equals the old policy, stop, because
then an optimal policy has been found. Otherwise, redefine the old policy according to the new policy and go back to step 2.

Note that in our special case of only one subprocess, Equation [I] can be reduced to Equation [3]:

\[ g(\sigma) = \frac{f_\alpha(\sigma)}{h_\alpha(\sigma)}. \]  

### Stage and State Variables and Decisions

In the replacement model, the maximum age of a cow is 12 lactations, and the maximum calving interval is 17 mo, which results in a total of \(12 \times 17 = 204\) stages. In each stage, the cow is described by the following state variables (number of classes between brackets): production in current lactation \([(15); <74\%, 74\% \text{ to } 78\%, \ldots, 122\% \text{ to } 126\%, \text{ and } \geq 126\%]\), production in previous lactation \([(15); <74\%, 74\% \text{ to } 78\%, \ldots, 122\% \text{ to } 126\%, \text{ and } \geq 126\%]\), calving interval \([(18); 11 \ldots 17 \text{ mo and open cows}]\), clinical mastitis in current month \([(2); \text{yes or no}]\), accumulated number of mastitic quarters in current lactation up to and including previous month \([(4); 0, 1, 2, \text{ and } \geq 3]\), and accumulated number of mastitic quarters in previous lactation \([(4); 0, 1, 2, \text{ and } \geq 3]\). Production level is defined relative to cows of the same age and month of lactation in absence of genetic improvement and voluntary culling and corrected for expected calving interval and mastitis status. For a calving interval of 11 mo, the number of days open is assumed to be between 45.75 and 76.25. For each subsequent class, an additional 30.5 d are added to days open to reflect more accurately the prolonged calving interval. The last class is defined for open cows. Not all states are accessible at each stage: e.g., in mo 17 a cow can only be open or have a calving interval of 17 mo. Exclusion of those states that are not feasible results in an HMP model with 6,821,724 different states a cow may enter during her life.

The model optimizes three decisions that can be made at each state and stage: 1) keep the cow at least 1 mo more and do not inseminate her when in estrus (keep), 2) keep the cow at least 1 mo more and inseminate her when in estrus (insm), and 3) replace the cow immediately by a replacement heifer (repl).

Treatment of a cow is not defined as a separate decision. For each decision \(d\) at each state \(i\) and stage \(t\), the HMP algorithm (see Equation [2]) generates a relative value of expected net revenues, assuming optimal decisions in the future. With those relative values \(V_i(t,d)\), the impact of the decision can be evaluated by two key figures: retention pay-off (RPO) (also termed future profitability) and insemination value (IV) (25). Subscript \(\alpha\) is omitted because in our case only one subprocess is defined.

\[ \text{RPO}_i(t) = \max(V_i(t,\text{keep}),V_i(t,\text{insm})) - V_i(t,\text{repl}) \]

\[ \text{IV}_i(t) = V_i(t,\text{insm}) - V_i(t,\text{keep}). \]

Thus, retention pay-off is the total extra profit to be expected in the future from keeping (keep) or inseminating (insm) a cow until she reaches her optimal lifespan, compared with immediate replacement (repl), taking into account the risk of involuntary disposal of retained cows (10). The IV is the extra profit to be expected in the future from inseminating a cow, compared with leaving her open for at least 1 mo more, taking into account the risk of no conception and involuntary disposal. Keep and replace decisions have to be made in all states. Insemination is defined as possible between 3 to 9 mo in lactation only (i.e., calving interval of 11 through 17 mo). In the model the decision to replace results in an immediate replacement (i.e., at the beginning of the month). Cows are kept at least 1 mo more when the decision to keep or to inseminate has been chosen. Involuntary replacement (e.g., due to lameness and death) can occur at the end of each month.

The components of HMP to be defined further are (see also steps 2 and 3 of the optimization algorithm) the immediate expected rewards \(r_i(t,d)\), physical output \(m_i(t,d)\), and the transition probabilities \(p_i(t,d)\) and \(p_i(0)\). Subsequent sections describe how the decision and stage-dependent immediate expected rewards are calculated, using a gross margin model.

### Gross Margin Model

The model that calculates gross margins from milk production, calf sales, feed costs, and sundry costs was described by Van Arendonk (23). Regular fixed cost of labor supplied
TABLE 1. Basic prices and other parameters used in determining the optimal replacement policy.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milk fat, /kg</td>
<td>4.72</td>
</tr>
<tr>
<td>Milk protein, /kg</td>
<td>7.78</td>
</tr>
<tr>
<td>Base price of milk, /100 kg</td>
<td>-1.61</td>
</tr>
<tr>
<td>Female calves, /kg</td>
<td>3.67</td>
</tr>
<tr>
<td>Male calves, /kg</td>
<td>5.86</td>
</tr>
<tr>
<td>Roughages, /MJ of NE</td>
<td>0.021</td>
</tr>
<tr>
<td>Concentrates, /MJ of NE</td>
<td>0.028</td>
</tr>
<tr>
<td>Carcass weight, /kg</td>
<td>3.33</td>
</tr>
<tr>
<td>Price of replacement heifer</td>
<td>1444</td>
</tr>
<tr>
<td>Insemination</td>
<td>11</td>
</tr>
<tr>
<td>Mature equivalent (8 yr) herd level</td>
<td>7750</td>
</tr>
<tr>
<td>Milk, kg</td>
<td></td>
</tr>
<tr>
<td>Fat content, %</td>
<td>4.35</td>
</tr>
<tr>
<td>Protein content, %</td>
<td>3.39</td>
</tr>
<tr>
<td>Other</td>
<td></td>
</tr>
<tr>
<td>Age at first calving, mo</td>
<td>24</td>
</tr>
<tr>
<td>Mature live weight, kg</td>
<td>650</td>
</tr>
</tbody>
</table>

1For a heifer 7 mo in lactation.

by the farmer was not included. In our study, housing costs were not included either and were considered to be fixed costs. Groen (7) and Jalvingh et al. (11) slightly modified this gross margin model and updated the prices. Those modifications were also incorporated in our model.

For each month in lactation the gross margins from milk production were determined and based on fat and protein contents. Feed costs were calculated from consumption of roughage and concentrates, estimated from the energy requirements. Furthermore, calf revenues were included. Parameters of, and prices in, the model were chosen to represent the Black and White cows in The Netherlands at normalized price levels of 1989 to 1991. In Table 1, the basic prices and other parameters used in the gross margins model are shown.

The gross margin model of Van Arendonk (23) was extended to include effects of clinical mastitis. Houben et al. (8) estimated the effects of clinical mastitis on production. Major results of that study were used to determine normalized monthly production effects from accumulated number of clinical quarter cases in current lactation up to and including the previous month (CQ) and the accumulated number of clinical quarter cases in the previous lactation (PQ). The effect of clinical mastitis in the current month (DQ) was estimated indirectly, because Houben et al. (8) had concluded that this effect was underestimated in their study.

As can be concluded from the work of Houben et al. (8), the typical pattern of the CQ effect and, to a lesser extent, the PQ effect on accumulated production showed that accumulated production losses increased asymptotically until about 10 mo in lactation. Most of the mastitis cases occur at the beginning of the lactation, and those cases only have a minor effect on production later in the lactation. Estimates of production losses by CQ and PQ are normalized by assuming that the maximum accumulated production losses are observed in mo 10 and then stay at that level and that production losses reach that maximum according to a second-degree polynomial, which starts in mo 1 with an accumulated production loss of 0 kg. With those assumptions, calculation of the monthly production losses for each CQ (0, 1, 2, and 23) and PQ (0, 1, 2, and 23) was possible. Different parameters were used for the first lactation than for second and later lactations. Table 2 shows the maximum production losses for each CQ and PQ for lactations 1 and 22. Milk production losses caused by ≥3 clinical cases in current lactation were 132 kg in the first lactation and 544 kg in the second and later lactation (Table 2). One or 2 clinical cases in the previous lactation had only a minor effect on the production in current lactation (PQ in Table 2).

For background information and more details about the data in Table 2, the reader is referred to Houben et al. (8).

In the gross margin model, a multiplicative effect of mastitis was used, and, therefore, the normalized monthly milk production losses were multiplied by a correction factor (corr) to accomplish this multiplicative effect.

\[
\text{corr} = \left( \frac{\text{RelProd}}{100} \right) \left( \frac{305\text{Prod}}{305\text{Ref}} \right)
\]

where

\[
\text{corr} = \text{correction factor to obtain a multiplicative effect,}
\]

RelProd = relative production of a cow (percentage),

TABLE 2. Maximum accumulated production losses until mo 10 by number of clinical quarter cases in current lactation (CQ) and in previous lactation (PQ).

<table>
<thead>
<tr>
<th></th>
<th>CQ</th>
<th>PQ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Lactation 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Milk, kg</td>
<td>0</td>
<td>47</td>
</tr>
<tr>
<td>Fat, kg</td>
<td>0</td>
<td>5.1</td>
</tr>
<tr>
<td>Protein, kg</td>
<td>0</td>
<td>1.1</td>
</tr>
<tr>
<td>Lactation ≥2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Milk, kg</td>
<td>0</td>
<td>166</td>
</tr>
<tr>
<td>Fat, kg</td>
<td>0</td>
<td>6.9</td>
</tr>
<tr>
<td>Protein, kg</td>
<td>0</td>
<td>4.5</td>
</tr>
</tbody>
</table>

305Prod$_1$ = average 305-d milk production according to the gross margin model in lactation 1,

305Ref$_1$ = average 305-d milk production of cows in lactation 1 (4824, 5724, and 6215 kg for lactations 1, 2, and ≥3, respectively) in the study by Houben et al. (8).

The production losses from DQ were calculated indirectly. According to Morris and Marsh (18), several studies produced loss estimates to average 10% or more, assuming that each infected cow has, on average, one to two infected quarters. To obtain approximately the same production losses, we accordingly assumed for each clinical quarter case that 1) milk production in the month after mastitis had occurred was reduced by 40% in lactation 1 and by 50% in later lactations, 2) fat production was reduced by 45% in lactation 1 and 55% in later lactations, and 3) protein production by 30% in lactation 1 and 40% in later lactations.

According to the data used by Houben et al. (8), the average number of clinical cases per cow case is 1.29 in mo 1 to 4 and 1.18 in mo ≥5. Consideration of this information combined with the estimated effect of DQ and CQ on production led to an expected production loss for each cow case. Table 3 shows the expected total production loss in a lactation and relative production loss for a cow case diagnosed in a certain month and the percentage of the total production loss that appears in the month of diagnosis, for a cow with an average production and a calving interval of 12 mo. A clinical cow case in the 1st or 2nd mo of lactation ≥2 reduced milk production per lactation by about 10%, which agrees with the findings of Morris and Marsh (18) (Table 3). At between 79 and 85% of production, losses occurred in the month of infection (percentages of DQ in Table 3). In the first lactation, a maximum reduction in lactation milk production was observed when a cow contracted mastitis in the 2nd mo (7.3%). When mastitis occurred in mo ≥7, >95% of the production losses were observed in the month of infection (Table 3).

Other effects of clinical mastitis included in the gross margin model are treatment costs, costs of discarded milk, and positive effect on feed consumption.

Treatment Costs

All cows with clinical mastitis that were not replaced voluntarily were assumed to have been treated according to the following distribution (based on data used by Houben et al. (8)): milk stripping (1.2%), intramammary (31.5%), parenteral (3.6%), and both intramammary and parenteral (63.7%). For each intramammary treatment, three injectors were used at a price of US$1.94 (US$1 = 1.80Dfl.) each, and each parenteral treatment cost US$14. It was assumed that a veterinarian had to visit the farm for 25% of the clinical quarter cases at a cost of US$22 per visit. Furthermore, a farmer was assumed to spend 2 h for each new quarter case at a price of US$15.30/h (note that only additional labor costs were
included). Of the quarter cases, 8.4% occurred in the same cow on the same day, and, therefore, only 91.6% of the mastitic quarters needed extra labor and veterinary visits. These figures led to average treatment costs for each clinical quarter of US$49 (only in case a cow was not culled).

**Costs of Discarded Milk**

Production losses from clinical mastitis do not only occur from reduced milk production but also because milk with antibiotics cannot be delivered to the milk factory. The number of days of no delivery depends on treatment: milk stripping (0 d), intramammary (6 d), parenteral (4 d), and both intramammary and parenteral (6 d). However, the cost of discarded milk is less than the normal value of milk because this milk can partly be used for calves, replacing milk powder. To ensure quality, milk could not be used to replace milk powder in the first 1.5 d, and thus had no value during that period. The actual production of a cow was used (i.e., lower production because of mastitis) to determine the value and the alternative value of the discarded milk. The alterna-

<table>
<thead>
<tr>
<th>Lactation and month</th>
<th>Milk Loss (%)</th>
<th>%DQ</th>
<th>Milk Loss (%)</th>
<th>%DQ</th>
<th>Milk Loss (%)</th>
<th>%DQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lactation 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 mo</td>
<td>394</td>
<td>6.8</td>
<td>87.6</td>
<td>22.5</td>
<td>8.8</td>
<td>77.8</td>
</tr>
<tr>
<td>2 mo</td>
<td>420</td>
<td>7.3</td>
<td>90.9</td>
<td>22.2</td>
<td>8.7</td>
<td>82.3</td>
</tr>
<tr>
<td>3 mo</td>
<td>392</td>
<td>6.8</td>
<td>92.7</td>
<td>20.4</td>
<td>8.0</td>
<td>85.4</td>
</tr>
<tr>
<td>4 mo</td>
<td>364</td>
<td>6.3</td>
<td>94.3</td>
<td>18.6</td>
<td>7.3</td>
<td>88.5</td>
</tr>
<tr>
<td>5 mo</td>
<td>310</td>
<td>5.4</td>
<td>95.4</td>
<td>16.0</td>
<td>6.2</td>
<td>90.2</td>
</tr>
<tr>
<td>6 mo</td>
<td>287</td>
<td>5.0</td>
<td>96.8</td>
<td>14.9</td>
<td>5.8</td>
<td>93.3</td>
</tr>
<tr>
<td>7 mo</td>
<td>264</td>
<td>4.6</td>
<td>98.1</td>
<td>13.7</td>
<td>5.4</td>
<td>95.3</td>
</tr>
<tr>
<td>8 mo</td>
<td>237</td>
<td>4.1</td>
<td>99.1</td>
<td>12.6</td>
<td>4.9</td>
<td>97.5</td>
</tr>
<tr>
<td>9 mo</td>
<td>195</td>
<td>3.4</td>
<td>99.7</td>
<td>10.6</td>
<td>4.1</td>
<td>99.1</td>
</tr>
<tr>
<td>10 mo</td>
<td>143</td>
<td>2.5</td>
<td>100.0</td>
<td>7.9</td>
<td>3.1</td>
<td>100.0</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Lactation 2</th>
<th></th>
<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>1 mo</td>
<td>699</td>
<td>10.3</td>
<td>78.9</td>
<td>34.6</td>
<td>11.4</td>
<td>79.2</td>
</tr>
<tr>
<td>2 mo</td>
<td>705</td>
<td>10.4</td>
<td>83.5</td>
<td>33.3</td>
<td>11.0</td>
<td>83.0</td>
</tr>
<tr>
<td>3 mo</td>
<td>641</td>
<td>9.4</td>
<td>86.2</td>
<td>30.2</td>
<td>10.0</td>
<td>85.9</td>
</tr>
<tr>
<td>4 mo</td>
<td>581</td>
<td>8.5</td>
<td>88.9</td>
<td>27.4</td>
<td>9.1</td>
<td>89.0</td>
</tr>
<tr>
<td>5 mo</td>
<td>482</td>
<td>7.1</td>
<td>90.7</td>
<td>23.1</td>
<td>7.5</td>
<td>90.9</td>
</tr>
<tr>
<td>6 mo</td>
<td>433</td>
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<td>93.4</td>
<td>21.1</td>
<td>7.0</td>
<td>93.8</td>
</tr>
<tr>
<td>7 mo</td>
<td>386</td>
<td>5.7</td>
<td>95.8</td>
<td>19.2</td>
<td>6.3</td>
<td>96.4</td>
</tr>
<tr>
<td>8 mo</td>
<td>335</td>
<td>4.9</td>
<td>97.9</td>
<td>17.0</td>
<td>5.6</td>
<td>98.3</td>
</tr>
<tr>
<td>9 mo</td>
<td>266</td>
<td>3.9</td>
<td>99.3</td>
<td>14.0</td>
<td>4.6</td>
<td>99.9</td>
</tr>
<tr>
<td>10 mo</td>
<td>189</td>
<td>2.8</td>
<td>100.0</td>
<td>10.1</td>
<td>3.3</td>
<td>100.0</td>
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</table>

<table>
<thead>
<tr>
<th>Lactation 3</th>
<th></th>
<th></th>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mo</td>
<td>788</td>
<td>10.7</td>
<td>80.6</td>
<td>38.8</td>
<td>11.8</td>
<td>80.8</td>
</tr>
<tr>
<td>2 mo</td>
<td>780</td>
<td>10.6</td>
<td>84.6</td>
<td>36.8</td>
<td>11.2</td>
<td>84.0</td>
</tr>
<tr>
<td>3 mo</td>
<td>705</td>
<td>9.5</td>
<td>87.1</td>
<td>33.2</td>
<td>10.1</td>
<td>86.6</td>
</tr>
<tr>
<td>4 mo</td>
<td>635</td>
<td>8.6</td>
<td>89.5</td>
<td>29.9</td>
<td>9.1</td>
<td>89.3</td>
</tr>
<tr>
<td>5 mo</td>
<td>522</td>
<td>7.1</td>
<td>91.2</td>
<td>25.1</td>
<td>7.7</td>
<td>90.9</td>
</tr>
<tr>
<td>6 mo</td>
<td>464</td>
<td>6.3</td>
<td>93.6</td>
<td>22.6</td>
<td>6.9</td>
<td>93.4</td>
</tr>
<tr>
<td>7 mo</td>
<td>407</td>
<td>5.5</td>
<td>95.9</td>
<td>20.4</td>
<td>6.2</td>
<td>95.7</td>
</tr>
<tr>
<td>8 mo</td>
<td>347</td>
<td>4.7</td>
<td>97.9</td>
<td>17.8</td>
<td>5.4</td>
<td>97.4</td>
</tr>
<tr>
<td>9 mo</td>
<td>269</td>
<td>3.6</td>
<td>99.3</td>
<td>14.2</td>
<td>4.3</td>
<td>99.2</td>
</tr>
<tr>
<td>10 mo</td>
<td>186</td>
<td>2.5</td>
<td>100.0</td>
<td>10.0</td>
<td>3.1</td>
<td>100.0</td>
</tr>
</tbody>
</table>

1Based on mean production (within cow class) and calving interval of 12 mo; month = month within lactation number, loss = expected losses in kilogram and as percentage of lactation production, and %DQ = percentage that appears in month of diagnosis.
The value of discarded milk was set at US$.17/kg.

**Effect on Feed Consumption**

Mastitis reduces milk production, and, hence, less feed consumption is necessary. The reduced energy need because of less milk, fat, and protein production was considered in the gross margin model. Milk was assumed to be produced with the same efficiency as by healthy cows.

**Immediate Expected Reward**

With the information from the gross margin model, the immediate expected rewards \( r_i(t,d) \) for state \( i \) (a combination of production, production in previous lactation, calving interval, DQ, CQ, and PQ) at stage \( t \) (a combination of lactation and month in lactation) and decision \( d \) (keep, inseminate, or replace) were calculated as follows:

\[
GM_i(t) - HC - VC(t) + [pIR(t) \times (SE_i(t) - RCV(t))],
\]

\[
t = 1, \ i \neq I
\]

\[
GM_i(t) - VC(t) + [pIR(t) \times (SE_i(t) - RCV(t))],
\]

\[
2 \leq t \leq T - 1, \ t \neq CI, \ i \neq I
\]

\[
GM_i(t) - VC(t) + [pIR(t) \times (SE_i(t) - RCV(t))]
\]

\[
+ [(1 - pIRN) \times PE_i(t)],
\]

\[
t = CI, \ i \neq I
\]

\[
GM_i(t) - VC(t) + SE_i(t)
\]

\[
- [pIR(t) \times RCV(t)],
\]

\[
t = T, \ i \neq I
\]

\[
0, \ i = I
\]

where

\( r_i(t, \text{keep}) \) = immediate expected reward for state \( i \) at stage \( t \) and decision is to keep the cow for at least one more month,

\( GM_i(t) \) = gross margin for state \( i \) at stage \( t \),

\( HC \) = costs of replacement heifer,

\( VC(t) \) = veterinary costs at stage \( t \),

\( pIR(t) \) = probability of involuntary replacement at stage \( t \) (see also section about transition probabilities),

\( SE_i(t) \) = carcass value for state \( i \) at end of stage \( t \),

\( RCV(t) \) = reduction in carcass value because of involuntary culling,

\( pIRN \) = total probability of involuntary replacement during next lactation,

\( PE_i(t) \) = production effect of length of calving interval,

\( CI \) = stage in which month is equal to calving interval,

\( T \) = last month in last lactation, and

\( I \) = replacement state.

Equation [5] shows that in the last month of a lactation \( t = CI \), the production effect of the length of calving interval, weighed for the probability of realization of next lactation, is added to the immediate expected reward. It would have been preferable theoretically to add this effect in the next lactation. However, this month is the last in which the calving interval is known.

Regular veterinary costs \( VC \) were obtained from Van Arendonk (23): US$17, 6, 6, and 3 in 1, 2, 3, and ≥3 lactations, respectively.

\[
r_i(t, \text{insm}) = \begin{cases} 
GM_i(t) - VC(t) - IC + [pIR(t) \times (SE_i(t) - RCV(t))], & 0 \leq t \leq LI, \ i \neq I \\
0, & i = I 
\end{cases}
\]

where

\( r_i(t, \text{insm}) \) = immediate expected reward for state \( i \) at stage \( t \) and decision is to inseminate the cow and to keep her for at least 1 mo more,

\( IC \) = insemination costs,

\( FI \) = stage in which month is month of first insemination (i.e., stage 3), and

\( LI \) = stage in which month is month of last insemination (i.e., stage 9)

where

\[
r_i(t, \text{repl}) = \begin{cases} 
SE_i(t - 1), & 1 \leq t \leq T, \ i \neq I \\
0, & i = I 
\end{cases}
\]

\( r_i(t, \text{repl}) \) = immediate expected reward for state \( i \) at stage \( t \) and decision is to replace the cow immediately with a replacement heifer.
TABLE 4. Marginal probabilities of replacement.

<table>
<thead>
<tr>
<th>Lactation</th>
<th>Total marginal probability (A)</th>
<th>Because of</th>
<th>Udder health or mastitis (C)</th>
<th>Marginal probability of involuntary culling (A - B - C - D)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Production (B)</td>
<td>Reproduction (C)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>21</td>
<td>7</td>
<td>2.2</td>
<td>2.1</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>5</td>
<td>2.5</td>
<td>3.0</td>
</tr>
<tr>
<td>3</td>
<td>21</td>
<td>3</td>
<td>3.6</td>
<td>4.0</td>
</tr>
<tr>
<td>4</td>
<td>22</td>
<td>2</td>
<td>5.0</td>
<td>4.8</td>
</tr>
<tr>
<td>5</td>
<td>23</td>
<td>0</td>
<td>6.3</td>
<td>5.7</td>
</tr>
<tr>
<td>6</td>
<td>25</td>
<td>0</td>
<td>7.8</td>
<td>6.7</td>
</tr>
<tr>
<td>7</td>
<td>27</td>
<td>0</td>
<td>9.0</td>
<td>7.4</td>
</tr>
<tr>
<td>8</td>
<td>29</td>
<td>0</td>
<td>10.1</td>
<td>7.8</td>
</tr>
<tr>
<td>9</td>
<td>32</td>
<td>0</td>
<td>11.3</td>
<td>7.8</td>
</tr>
<tr>
<td>10</td>
<td>35</td>
<td>0</td>
<td>12.2</td>
<td>7.7</td>
</tr>
<tr>
<td>11</td>
<td>38</td>
<td>0</td>
<td>13.0</td>
<td>7.5</td>
</tr>
<tr>
<td>12</td>
<td>41</td>
<td>0</td>
<td>13.6</td>
<td>7.3</td>
</tr>
</tbody>
</table>

Physical Output

Within the HMP approach the physical output has to be defined. Physical output is the denominator of the object function (see Equation [3]). In our study, in which the optimization criteria were to maximize gross margin per time unit, length of a stage (time) is in HMP terms the immediate physical output: \( m_t(t, d) \). Time unit is 30.5 d (1 mo). From the definition of each decision, follows:

\[
m_t(t, \text{keep}) = \begin{cases} 1, & 1 \leq t \leq T, i \neq I \\ 0, & i = I \end{cases} \quad [8]
\]

\[
m_t(t, \text{insm}) = \begin{cases} 1, & I \leq t \leq L1, i \neq I \\ 0, & i = I \end{cases} \quad [9]
\]

\[
m_t(t, \text{repl}) = 0, \quad 1 \leq t \leq T, \quad 1 \leq i \leq I \quad [10]
\]

Transition Probabilities

The model was applied to situations in which cullings for age, low production, fertility status, and mastitis state were incorporated in the decision-making process of the HMP, which were also the stochastic elements of the model. The next state of the cow depended on the current state, the current stage, the decision to keep or to cull, the probability of conception if inseminated, the probability of survival to the next stage, the probability of transition to a different production, and the probability of clinical mastitis.

The type of cow disposal not subject to decision-making processes was referred to as involuntary (23). The total marginal probabilities of disposal and the marginal probabilities of disposal because of production, reproduction, and udder health or mastitis were taken from Dijkhuizen (4). From those figures the marginal probability of involuntary disposal was calculated for lactations 1 to 12 (Table 4).

The proportions of disposal during each month of lactation are 20, 8, 7, 7, 7, 7, 7, 6, and 5% for mo 1 to 12, respectively, and 4% for higher months (23). The calculation of the probability of conception for mo 3 to 9 was obtained from Van Arendonk and Dijkhuizen (26) and modified by Jalvingh et al. (11). Rate of detection of estrus was set at 70%.

One of the objectives of the present study was to develop a DP model allowing monthly transitions to other production levels. Dommernholt (5) found a coefficient of variation of 12% for lactation production, after correction for the effects of age and herd-year-season. In our study, we assumed a constant variation coefficient of accumulated production throughout the lactation. Using correlations between the milk production in the current and next month and between the milk production in the previous lactation and the next month, multiple regression factors and reliabilities of the transition probabilities were calculated analogously...
to those of Van Arendonk (23). With those multiple regression factors, the accumulated production for the next month was estimated from the accumulated production in the current month and previous lactation. Correlations were calculated from data used by Houben et al. (8). The correlations between the accumulated milk production in the next month and current month were 0.956, 0.979, 0.988, 0.991, 0.993, 0.994, 0.996, 0.996, and 0.997 for, respectively, mo 1 to 9 and 1 for mo ≥10. The correlations between the accumulated milk production in the next month and mo 10 in previous lactation were 0.391, 0.429, 0.478, 0.498, 0.535, 0.544, 0.545, and 0.547 for mo 1 to 8 and 0.55 for mo ≥9. The correlation between the 1st mo in a lactation and last month in previous lactation was 0.327. Those correlations resulted in a repeatability of lactation production of 0.55 and 0.42 for a one- and two-lactation interval, respectively, which closely agrees with results of Van Arendonk (23).

To reduce the amount of calculations needed during the optimization procedure, transitions were pruned when the transition probability was <0.05 times the reliability of the regression factors in that month. For the cow replacement model with typical high repeatabilities between monthly accumulated production, the pruning factor reduced the amount of calculations by almost 85%; 99% of all transitions were still covered.

The coefficients that calculate the probability that a cow will contract clinical mastitis in the next stage was calculated according to the Equation [11]:

\[
p(t + 1) \mid i(t), d(t) = \begin{cases} 
\frac{\exp(f_{t}(0))}{1 + \exp(f_{t}(0))} & \text{if } t = 1, \ldots, CI - 2 \\
\frac{30.5}{23.5} \times \exp(f_{t}(t))}{1 + \frac{30.5}{23.5} \times \exp(f_{t}(t))} & \text{if } t = CI
\end{cases}
\]

where

\[p(t + 1) \mid i(t), d(t)\]

= conditional probability of at least one clinical quarter in stage t + 1, given current state i(t) and decision d(t);

\[f_{t}(0)\]

= function of logistic regression coefficients obtained from Houben et al. (8) for later months in lactation. Risk factors in this function are clinical mastitis in current month, in previous months, and, in previous lactation, lactation number, month in lactation, and production;

\[pDP_{q0}\]

= probability of mastitis in last month of dry period depending on occurrence of mastitis in month before last month in dry period (q(t)); and

\[f_{t}(t)\]

= function of logistic regression coefficients obtained from Houben et al. (8) for 1st mo in lactation, and risk factors in this function are clinical mastitis in current month and in previous months.

As can be seen in Equation [11], a correction was made for the probability of mastitis in the 1st mo of lactation because those coefficients were based on 23.5 d instead of 30.5 d.

Based on results of Houben et al. (8), the following distribution of clinical cases in a month during which mastitis was observed was
TABLE 5. The expected farm results when basic parameters are used in contrast to results for alternative parameters.1

<table>
<thead>
<tr>
<th></th>
<th>Basic</th>
<th>$r_p = 0$</th>
<th>$r_p = .5$</th>
<th>$r_p = 2$</th>
<th>$r_l = .5$</th>
<th>$r_l = 2$</th>
<th>notr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross margin, US$/yr per cow</td>
<td>2431</td>
<td>83</td>
<td>48</td>
<td>-123</td>
<td>24</td>
<td>-28</td>
<td>10</td>
</tr>
<tr>
<td>Milk production, kg of FPCM2yr per cow</td>
<td>7735</td>
<td>168</td>
<td>96</td>
<td>-219</td>
<td>77</td>
<td>-78</td>
<td>58</td>
</tr>
<tr>
<td>Feed costs, US$/yr per cow</td>
<td>896</td>
<td>14</td>
<td>8</td>
<td>-19</td>
<td>6</td>
<td>-6</td>
<td>4</td>
</tr>
<tr>
<td>Calves, no./yr per cow</td>
<td>1.14</td>
<td>-0.1</td>
<td>-0.1</td>
<td>.05</td>
<td>0</td>
<td>.02</td>
<td>.02</td>
</tr>
<tr>
<td>Calving interval, d</td>
<td>370</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Calving interval ≥14 mo, %</td>
<td>11.8</td>
<td>3</td>
<td>2</td>
<td>-3</td>
<td>0</td>
<td>.1</td>
<td>.4</td>
</tr>
<tr>
<td>Mastitis incidence/10,000 cow days</td>
<td>9.7</td>
<td>-9.7</td>
<td>-5.4</td>
<td>10.0</td>
<td>.6</td>
<td>-1.0</td>
<td>.2</td>
</tr>
<tr>
<td>Replacement rate, %/yr</td>
<td>29.2</td>
<td>-2.1</td>
<td>-1.5</td>
<td>8.4</td>
<td>-1.0</td>
<td>1.5</td>
<td>2.4</td>
</tr>
<tr>
<td>Voluntary replacement, %/yr</td>
<td>18.3</td>
<td>-2.2</td>
<td>-1.6</td>
<td>8.5</td>
<td>-1.1</td>
<td>1.5</td>
<td>2.4</td>
</tr>
<tr>
<td>Voluntary replacement with mastitis, %/yr</td>
<td>3.7</td>
<td>-3.7</td>
<td>-2.5</td>
<td>9.0</td>
<td>-1.5</td>
<td>2.9</td>
<td>0.1</td>
</tr>
</tbody>
</table>

1$rp =$ Relative probability of mastitis, $rl =$ relative production losses caused by clinical mastitis, and notr = no transitions of production level within lactation are assumed.
2FPCM = Fat- and protein-corrected milk. FPCM = .349 milk (kilograms) + 10.7 fat (kilograms) + 6.7 protein (kilograms).
3Values represent increase or decrease in basic model parameters.

The gross margin model and optimization model were written in Pascal and run on a SUN SparcStation 1 (Sun Microsystems, Inc. using UNIX). The program first calculates the gross margins for all states, which takes about 10 min of computation time, and then starts with the optimization, which takes about 6 h of computation time.

This section shows the effect of using the optimal replacement and insemination policy on the farm results in the basic situation. Furthermore, the effect of the state of a cow on the retention pay-off and insemination value are shown. To gain more insight into major model characteristics, a sensitivity analysis was carried out.

**Basic Results**

The optimal replacement and insemination policy in the basic farm resulted in a replacement rate of 29.2% annually, of which 18.3% was voluntary (Table 5). In the basic situation, the model supported decisions related to 63% of all replacements. Furthermore, 12.7% (3.7 of 29.2) of the culled cows were infected with clinical mastitis at culling (i.e., 20.2% of voluntarily culled cows). This result does not mean, however, that mastitis was the only reason for culling. The gross margin in the basic situation was US$2431/yr per cow (housing and fixed labor costs not included), and feed costs were US$896/yr per cow. Milk production corrected for fat and protein was 7735 kg/yr per cow. Mastitis occurred at an...
incidence rate of 9.7 clinical cases per 10,000 cow days. Table 5 also shows results when no mastitis occurred (relative probability = 0: the gross margin increased by US$83/yr per cow, the total loss from mastitis.

Changes in Incidence of Clinical Mastitis

Changes of the probability that clinical mastitis will occur had a major effect on the farm results. A 50% reduction of this probability (rp = .5) increased the gross margin by US$48/yr per cow and, when the probability of contracting mastitis had doubled (rp = 2), the gross margin decreased by US$123/yr per cow (Table 5). An increase of risk of mastitis infection leads to much more voluntary culling (+8.5%) according to the optimal policy. In turn, the high replacement rate had a strong negative effect on the gross margin. Although many of the mastitic cows were culled when probability of mastitis was high (47% of all voluntarily replaced cows had clinical mastitis), the mastitis incidence had still doubled. Therefore, the effect of culling on mastitis incidence was minimal, probably because replacement heifers also run a high risk of mastitis infection. Table 5 shows that the relationship is linear between relative risk of mastitis (alternative rp) and mastitis incidence and, to a lesser extent, also between relative risk and gross margin. Because of the high rate of involuntary culling, according to the optimal policy for high relative risk of mastitis, the gross margin was expected to have been relatively more reduced. The relative risk of mastitis had a small effect on the insemination decisions, which can be inferred from the few changes in average calving interval and percentage of cows with a calving interval of >13 mo.

Changes in Production Losses Caused by Clinical Mastitis

The effect of changing production losses caused by clinical mastitis is presented in Table 5. In contrast to the relative probability of clinical mastitis, the relative losses caused by clinical mastitis had no linear effect on the gross margin. Apparently culling was an effective way to reduce the losses relatively because the decrease in gross margin when production losses were increased by 100% (Table 5; relative loss = 2) was only slightly higher than the increase in gross margin for the alternative when production losses were reduced by 50% (relative loss = .5). So, in contrast to relative probability, relative production losses did not have a linear effect on gross margin. Doubled relative production losses resulted in 33.3% of the voluntarily replaced cows having clinical mastitis in contrast to 20.2% in the basic situation and 12.8% in the situation in which production losses had been halved. Relative production losses had an even smaller effect on the insemination decisions as relative risk. Calving interval and percentage of cows with a calving interval of >13 mo hardly changed.

Changes Within-Lactation Transitions of Production Class

To find out the effect of including within-lactation transitions of production class on farm results, an alternative situation in which those transitions were not included was defined (Table 5; without transitions). The repeatability of total lactation production for one and two lactation intervals was kept at the same level (.55 and .42, respectively). The most remarkable finding for the alternative, without transitions within lactation, was the increase in voluntary replacement rate by 2.4%. Because the percentage of the voluntarily replaced cows with clinical mastitis was almost the same as in the basic situation, it can be concluded that the increase of voluntary replacements was only for reasons related to production. Low producing cows were culled more frequently when no production transitions were allowed within lactation and when more cows had a calving interval of >13 mo. The latter effect may be caused because high producing cows remain longer in the herd. Despite the increase in voluntary replacement by 2.4%, the gross margin was increased by US$10, and milk production was corrected for fat and protein by 58 kg.

Calving Interval

In Figures 2 and 3, the effect of calving interval, month of gestation, and clinical mastitis state on retention pay-off are shown for cows in first and second lactations, respec-
Figure 2. Effect of calving interval (CI), month of gestation (GMONTH), and mastitis state on retention pay-off (RPO) for cows in first lactation. A cross means that cows should be culled anyway (i.e., retention pay-off <0), a cylinder means that cows should be kept, and a square pillar means that a cow should be replaced in case of clinical mastitis.

Square pillars in those figures mean that the optimal decision changed from keeping to culling in case of clinical mastitis; thus, it could be concluded that clinical mastitis did not have an effect on the replacement decisions for average producing pregnant cows with an expected calving interval ≤13 and 14 mo for lactations 1 and 2, respectively. Only 11.8% of the cows had calving intervals of >13 mo (Table 5), and, therefore, for most of the average or better producing pregnant cows, clinical mastitis did not have any effect on the optimal decision. It is economically optimal to replace pregnant first lactation cows immediately until the 5th mo of gestation (i.e., 14 mo in lactation) when the expected calving interval is 17 mo (Figure 2).

Relative Production

In Figures 4 and 5, the effect of relative production, month in lactation, and clinical mastitis state on retention pay-off and the insemination decision are shown for open cows in first and second lactations, respectively. Those cows had not had any clinical mastitis in the past, and the production in previous lactation was 100%.

It was economically optimal to replace open first lactation cows immediately in cases of clinical mastitis when production was below average (Figure 4). Open first lactation cows
Figure 5. Effect of relative production level (PROD in percentage), month in lactation (LMONTH), and mastitis state on retention pay-off (RPO) for open cows in second lactation. A cross means that cows should be culled anyway (i.e., retention pay-off <0), a cylinder means that cows should be kept, a square pillar means that a cow should be replaced in case of clinical mastitis, and a narrow cylinder or pillar means that it is optimal to leave the cow open at that moment (relative production level in first lactation was 100%). Producing below 78% were replaced anyway and, with production of <86%, first lactation cows were not inseminated again. Second lactation cows producing <86% were replaced immediately in cases of clinical mastitis, but not in the 1st mo of lactation. Delaying insemination for 1 mo was optimal for only a few cows. This situation is depicted with narrow cylinders or pillars in Figures 4 and 5 (in the model, insemination in the 1st and 2nd mo of lactation was not allowed). In general, the optimal economic situation was to inseminate a cow as soon as possible. Regardless of production, healthy cows were inseminated until mo 4 in second lactation. Insemination of high producing healthy cows at least until mo 9 in lactations 1 and 2 was most economical. Subsequently, cows should be replaced immediately after mo 9 or 10.

Clinical Mastitis in Previous Months

For average producing, open cows, the effect of clinical mastitis in previous months (CQ), month in lactation, and mastitis state in current month on RPO and insemination decision are shown in Figures 6 and 7.

First lactation cows with ≥2 clinical cases in previous months and 1 new quarter case in the current month were replaced immediately.
Figure 8. Effect of accumulated number of clinical quarter cases in previous lactation (FQ), month in lactation (LMONTH), and mastitis state on retention pay-off (Po) for open cows in second lactation. A cross means that cows should be culled anyway (i.e., retention pay-off < 0), a cylinder means that cows should be kept, a square pillar means that a cow should be replaced in case of clinical mastitis, and a narrow cylinder or square means that it is optimal to leave the cow open at that moment.

When a first lactation cow had ≥ 3 clinical cases in previous months but no mastitis in the current month, she was not inseminated again after mo 4 of lactation. However, the first lactation cow was not replaced before mo 8. First lactation cows were culled just 1 mo earlier, and cows in second lactation not at all, when clinical cases in previous months and no new quarter case had occurred.

Figure 8 shows that the effect of clinical quarter cases in previous lactation had only a very minor effect on the replacement decisions and no effect on the insemination decisions.

**DISCUSSION**

The strength of the model described in this paper is the integral evaluation of age, production, fertility, and mastitis aspects to support replacement and insemination decisions. Therefore, the model was able to support 63% of all replacement decisions.

The HMP approach proved to be very useful for large problems for optimization of replacements. In the present study, a DP model was developed, according to the HMP approach, with 6,821,724 unique states that a cow may enter during her life. Nevertheless, because most infeasible transitions were eliminated in advance, during the optimization run, an optimal policy could be determined in a reasonable computation time (optimization takes approximately 6 h on a SUN SparcStation 1). This computation time, however, hardly permits for expansion of the current model. Although the HMP approach is an important step forward for replacement optimization problems, HMP does not consider dimensionality, which may be solved by defining a model structure in which distinction is made between main effects (state variables) and interactions (between state variables). The HMP does not make this distinction, and, therefore, a major part of the computations is spent on interactions between state variables that are expected to have minor influence on the optimal policy only.

In dairy cow replacement models, with the exception of Kristensen's models (13, 15), transitions between different classes of milk production were assumed to have occurred only at the end of the lactation period. This assumption implied that the relative level of milk production performance remained the same throughout the entire lactation period. In our study, the effect was examined of inclusion of results for transitions within lactation on the farm. Results showed that more cows were replaced because of production (+2.4%) when no transitions within lactation were included in the model. The explanation of this effect is that, if no transitions were allowed within a lactation, the cows remained at the same production during a lactation. If transitions were allowed, the expected production of cows early in the lactation were close to average (less information), and later in lactation their final production level was reached, which was exactly as if no transitions were allowed (because the same total lactation production correlations were used). Consequently, high producing cows were overestimated, and low producing cows were underestimated, at the beginning of a lactation. Low producing cows, therefore, were culled too soon. Because most of the above average producing cows were already in the herd, decisions did not change for high producing cows. For correct justification of production capacity, replacement models ought to have production transitions within lactation.

Farm results under a policy that is optimal for production and reproduction decisions were for the basic situation and mainly in agreement.
with results of Jalvingh et al. (11), who used the same parameters in their gross margin model. The replacement rate was about 4\% higher in the model of Jalvingh et al. (11), which may be caused by exclusion of within-lactation transitions in their model and the use of higher correlations within lactation. The course of the retention pay-off was in agreement with research of Van Arendonk and Dijkstra (26).

Results showed that the support of the insemination decisions in the current model is only of importance with regard to the decision not to inseminate anymore. According to the optimal policy, cows were simply inseminated as soon as possible and only for low producing cows was it sometimes optimal to leave them open ≥1 mo and cull them afterward. The latter situation concerned only a few cows. The influence of mastitis state on replacement decisions was much bigger than on insemination decisions. The sensitivity analysis showed that mastitis parameters had hardly any effect on length of calving interval or on the percentage of cows with a calving interval of 13 mo, which means that those parameters had minor influence on the optimal insemination decisions. If seasonal effects were included, support of insemination decisions was expected to become more important because shifting the moment of insemination could become optimal.

In general, clinical mastitis in the current lactation, especially clinical mastitis in the current month, has a major influence on replacement decisions. However, in contrast to production, clinical mastitis concerns only a fraction of the total herd, and, therefore, the effects of mastitis-related parameters on the average gross margin were weakened.

The results showed that the number of PQ had only a small effect on the optimal replacement decision in spite of the high risk associated with mastitis in previous lactation. In the model, the relative risk of contracting mastitis in current lactation with incidence 1, 2, or >2 clinical cases in previous lactation was 2.0, 2.6, and 2.9, respectively. Those parameters and the ones that determine the production losses caused by clinical mastitis were taken from Houben et al. (8). They found that production losses related to clinical mastitis in previous lactation were only significant when three or more quarters were infected in previous lactation. Morris and Marsh (18) concluded in their study that an issue that had not been satisfactorily resolved was whether production remained decreased in the next lactation after an infection had been eliminated, or returned to normal, as both effects were found in other studies. The implemented small effect of production losses caused by clinical mastitis in previous lactation had, of course, its effect on the results, but it is still remarkable that the increased risk related to mastitis in previous lactation had only a small effect on the optimal policy. The model apparently can be reduced by a factor of 4 without affecting the optimal policy by excluding mastitis in previous lactation from the decision-making process.

The model does not focus on specific mastitis treatment decisions. Cows were assumed to have been always treated in cases of clinical mastitis when the decision was to keep or to inseminate the cow. Reasons to implement this assumption in such a way were the lack of reliable data on the effect of treatment on new mastitis infections and related production losses, and because in reality 99\% of the cows were treated parenterally, locally, or both (8). The bacteria that cause mastitis were not included in the decision-making process. A clinical quarter case was defined to be caused by a common bacterium. Bacteria that cause an infection with more severe production losses should in reality lead to earlier replacement than advised by the model. Because of model size limitations, it was not possible to include the bacteria directly in the replacement and insemination model.

Results are difficult to compare with those from the study in which clinical mastitis was included in a DP model (22) because, in their study, mastitis was a binomial state variable, and replacement decisions were taken only at the beginning of a lactation. Stott and Kennedy (22) stated in their discussion that culling of cows with mastitis is likely to reduce the risk of further infections in the herd but that the benefits of this effect were not included in their model. This statement is also valid for our model.

The results show that the value of the model for farms with a high incidence of mastitis or high production losses caused by mastitis was relatively high. When relative risk of mastitis had doubled, the number of cows per year per hundred that had mastitis at culling increased from 3.7 to 12.7. On the basis of
a doubled risk, an increase to approximately 7.4 was expected.

The sensitivity analysis showed a linear relationship between relative risk (in the range of 0 to 2) and mastitis incidence when the optimal policy was followed. Furthermore, an approximate linear relationship existed between mastitis incidence and gross margin. Additional calculations showed that, from those two relations, the break-even point for farm-level treatments was US$11/yr per cow for each unit reduction of clinical quarter case (in the range of 0 to 20 quarter cases per 10,000 cow days). For instance, a farm-level treatment (e.g., teat dipping), which reduces the number of clinical quarter cases per 10,000 cow days from 10 to 7.5, may cost US$27/yr at maximum per cow.

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